BHU

MATHEMATICS

SOLVED SAMPLE PAPER

* DETAILED SOLUTIONS

9001894070

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MAX.MARKS : 360 **MARKS SCORED :** **Time : 2 Hours**

INSTRUCTIONS

Attempt all 120 questions. Each question carries 3 marks. 1 negative mark for each wrong answer.

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1. The function f defined by $f(x) = (x + 2)e^{-x}$ is

(A) decreasing for all x

(B) decreasing on $(-\infty, -1)$ and increasing in $(-1,\infty)$

(C) Increasing for all x

(D) decreasing in $(-1,\infty)$ and increasing in $(-\infty, -1)$

2. The function $f(x) = x\sqrt{(ax - x^2)}, a < 0$

(A) Increases on the interval (0, 3a/4)

(B) decreases on the interval (3a/4, a)

(C) decreases on the interval (0,3a/4)

(D) increases on the interval (3a/4, a)

3. If $f''(x) > 0, \forall x \in R, f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4), 0 < x < \frac{\pi}{2}$, then g(x) is increasing in

(A) $\left(0, \frac{\pi}{4}\right)$ (B) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

(C) $\left(0, \frac{\pi}{3}\right)$ (D) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

4. If $f : R \to R$ is the function defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$, then

(A) f(x) is an increasing function

(B) f(x) is a decreasing function

(C) f(x) is onto (surjective)

(D) None of the above

5. If $f'(x)=|x|-\{x\}$, where {x} denotes the fractional part of x, then f(x) is decreasing in

(A) $\left(-\frac{1}{2},0\right)$ (B) $\left(-\frac{1}{2},2\right)$

(C) $\left(-\frac{1}{2},-2\right)$ (D) $\left(\frac{1}{2},\infty\right)$

6. If $f(x)=\left(\frac{x^a}{x^b}\right)^{a+b}\cdot\left(\frac{x^b}{x^c}\right)^{b+c}\cdot\left(\frac{x^c}{x^a}\right)^{c+a}$, then f'(x) is equal to

(A) 1 (B) 0

(C) x^{a+b+c} (D) None of these

7. If $f(x)=\left(\frac{\sin^m x}{\sin^n x}\right)^{m+n}\cdot\left(\frac{\sin^n x}{\sin^p x}\right)^{n+p}\cdot\left(\frac{\sin^p x}{\sin^m x}\right)^{p+m}$, then f'(x) then is equal to

(A) 0 (B) 1

(C) $\cos^{m+n+p}x$ (D) None of these

8. If $y=\left(x+\sqrt{1+x^2}\right)^n$, then $\left(1+x^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is

(A) n^2y (B) $-n^2y$

(C) $-y$ (D) $2x^2y$

9. If $\phi(x) = \log_5 \log_3 x$, then $\phi'(e)$ is equal to

(A) e log 5 (B) –e log 5

(C) $\frac{1}{e\log 5}$ (D) None of these

10. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and f'(x) = sin x^2, then $\frac{dy}{dx}$ is equal to

(A) $\sin\left(\frac{2x-1}{x^2+1}\right)^2 \cdot \left(\frac{2+2x+x^2}{\left(x^2+1\right)^2}\right)$

(B) $\sin\left(\frac{2x-1}{x^2+1}\right)^2 \cdot \left(\frac{2+2x-x^2}{\left(x^2+1\right)^2}\right)$

(C) $\sin\left(\frac{2x+1}{x^2+1}\right)^2 \cdot \left(\frac{2+2x-x^2}{\left(x^2+1\right)^2}\right)$

(D) None of these

11. If f(x) = $\log_x(\log x)$, then f'(x) at x = e is

(A) e

(B) $\frac{1}{e}$

(C) $\frac{2}{e}$

(D) 0

12. The function f (x) = x^x decreases on the interval

(A) (0,e)

(B) (0,1)

(C) (0,1/e)

(D) None of these

13. If $y = \sum_{r=1}^{x} \tan^{-1}\frac{1}{1+r+r^2}$ then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{1+x^2}$

(B) $\frac{1}{1+\left(1+x\right)^2}$

(C) 0

(D) None of these

14. If x = t^t and $y = t^{t^t}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{t^{t^t} \cdot \left[(1+\log t)\log t + \frac{1}{t}\right]}{(1+\log t)}$

(B) $\frac{t^{t^t}\left[1+\log t + \frac{1}{t}\right]}{(1+\log t)}$

(C) $\frac{t^{t^t}\left[(1+\log t)\log t + \frac{1}{t}\right]}{(1+\log t)^2}$

(D) None of these

15. The derivative of f(tan x) w.r.t. (sec x) at $x = \frac{\pi}{4}$, where f'(1) = 2 and $g'\left(\sqrt{2}\right) = 4$, is

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) 1

(D) None of these

16. If $\frac{d}{dx}\left(\frac{1+x^4+x^8}{1+x^2+x^4}\right) = ax^3 + bx$ then

(A) a = 4, b = 2

(B) a = 4, b = –2

(C) a = –2, b = 4

(D) None of these

17. If $f(x) = \cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \sin x \sin\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 3$ then (gof) (x) is equal to

(A) 1

(B) 2

(C) 3

(D) None of these

18. Let $f(x) = (x^3 + 2)^{30}$. If $f^n(x)$ is a polynomial of degree 20, where $f^n(x)$ denotes the nth derivative of f(x) w.r.t. x, then the value of n is

(A) 60 (B) 40

(C)70 (D) None of these.

19. Let $f(x) = 3x^2 + 4x\, g'(1) + g''(2)$ and $g(x) = 2x^2 + 3x\, f'(2) + f''(3)$ then

(A) $f'(1) = 22 + 12\, f'(2)$ (B) $g'(2) = 44 + 12\, g'(1)$

(C) $f''(3) + g''(2) = 10$ (D) All of the above

20. If $y = x^{n-1} \log x$, then $x^2y_2 + (3 - 2n)xy_1$ is equal to

(A) $-(n - 1)^2y$ (B) $(n - 1)^2y$

(C) $-n^2y$ (D) n^2y

21. If $2x = y^{1/5} + y^{-1/5}$, then $(x^2 - 1)y_2 + xy_1 = ky$ where k is equal to

(A) –25 (B) 25

(C) 16 (D) –16

22. If $y = [(\tan x)^{\tan x}]^{\tan x}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is equal to

(A) 1 (B) 2

(C) 0 (D) Nome of these

23. If $y = (1 + x)(1 + x^2)(1 + x^4)\ldots(1 + x^{2n})$, then $\frac{dy}{dx}$ at x = 0 is

(A) –1 (B) 1

(C) 0 (D) None of these

24. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x$, then $f'\left(\frac{\pi}{4}\right)$ is

(A) –1 (B) 2

(C) $\sqrt{2}$ (D) None of these

25. If $f(x) = \sin\left(\frac{\pi}{2}[x] - x^5\right), 1 < x < 2$ and [x] denotes the greatest integer less than or equal to x, then $f'\left(\sqrt[5]{\frac{\pi}{2}}\right)$ is equal to

(A) $5\left(\frac{\pi}{2}\right)^{4/5}$

(B) $-5\left(\frac{\pi}{2}\right)^{4/5}$

(C) 0

(D) None of these

26. The value of $\int_3^5 \frac{x^2}{x^2 - 4} dx$ is

(A) $2 - \log_e\left(\frac{15}{7}\right)$

(B) $2 + \log_e\left(\frac{15}{7}\right)$

(C) $2 + 4 \log_e 3 - 4 \log_e 7 + 4 \log_e 5$

(D) $2 - \tan^{-1}(15/7)$

27. $\left|\int_{10}^{19} \frac{\sin x \, dx}{1 + x^8}\right|$ is less than

(A) 10^{-10}

(B) 10^{-11}

(C) 10^{-7}

(D) 10^{-9}

28. The value of $\int_a^b \frac{|x|}{x} dx, a < b$ is

(A) b – a

(B) a – b

(C) b + a

(D) |b| – |a|

29. If (x) = a + bx + cx^2, then $\int_0^1 f(x)\,dx$ equals

(A) $\frac{1}{2}\left[f(0)+4f\left(\frac{1}{2}\right)+f(1)\right]$

(B) $\frac{1}{6}\left[f(0)+2f\left(\frac{1}{2}\right)+f(1)\right]$

(C) $\frac{1}{6}\left[f(0)+4f\left(\frac{1}{2}\right)+f(1)\right]$

(D) None of these

30. $\int_{-1}^{1}\left|(1-x)\right|dx$ equals

(A) –2

(B) 0

(C) 2

(D) 4

31. If $\int_0^1 \frac{dx}{2e^x-1}=p\log(qe-1)-r,$ then

(A) p = 1, q = 1, r = –1

(B) p = 1, q = 2, r = 1

(C) p = 1, q = 2, r = –1

(D) None of these

32. The smallest interval [a, b] such that $\int_0^1 \frac{dx}{\sqrt{1+x^4}}\in[a,b]$ is given by

(A) $\left[\frac{1}{\sqrt{2}},1\right]$

(B) [0, 1]

(C) $\left[\frac{1}{2},1\right]$

(D) $\left[\frac{3}{4},1\right]$

33. If f(x + y) = f(x). f(y) for all x, y where f'(0) = $k\neq 0,$ then f(x) can be expressed as

(A) ae^{kx}

(B) a cos kx + b sin kx

(C) kx

(D) None of these

34. The value of $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta \, d\theta$ is

(A) 0 (B) $\frac{1}{15}$

(C) 1 (D) $\frac{8}{3}$

35. If $\int_0^1 \frac{\tan^{-1} x}{x} dx$ equals

(A) $\int_0^{\pi/2} \frac{x}{\sin x} dx$ (B) $\frac{1}{2} \int_0^{\pi/2} \frac{x}{\sin x} dx$

(C) $\int_0^{\pi/2} \frac{\sin x}{x} dx$ (D) None of these

36. $\int_0^{\pi/3} \frac{\cos x}{3+4\sin x} dx = k \log\left(\frac{3+2\sqrt{3}}{3}\right)$, then k is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

37. The value of $\int_0^1 \left(1+e^{-x^2}\right) dx$ is

(A) –1 (B) 2

(C) $1 + e^{-1}$ (D) None of these

38. If $f(x) = ae^{2x} + be^x + cx$ satisfies the conditions $f(0) = -1$, $f'(\log 2) = 31$ and $\int_0^{\log 4} \left[f(x) - cx\right] dx = \frac{39}{2}$, then

(A) a = 5 (B) b = –5

(C) c = 2 (D) a = 3

39. Let a, b, c be non-zero real numbers such that

$$\int_0^1 \left(1+\cos^8 x\right)x\left(ax^2+bx+c\right)dx=\int_0^2 \left(1+\cos^8 x\right)$$ $(ax^2 + bx + c)$ dx, then the quadratic equation $ax^2 + bx + c = 0$ has

(A) no root in (0, 2)
(B) atleast one root in (1, 2)
(C) atleast one root in (0, 1)
(D) two imaginary roots

40. The value of $\int_{-1/2}^{1/2} \cos x \log \frac{1+x}{1-x} dx$ is

(A) 0
(B) $\frac{1}{2}$
(C) $\frac{-1}{2}$
(D) None of these

41. $\int_{-\pi/2}^{\pi/2} \sin^{10} x\left(6x^9-25x^7+4x^3-2x\right)dx$ equals

(A) π
(B) 0
(C) 25
(D) None of these

42. The value of $\int_{-\pi}^{\pi}\left(1-x^2\right)\sin x \cos^2 x \, dx$ is

(A) 0
(B) $\pi-\frac{\pi^3}{3}$
(C) $2\pi - \pi^3$
(D) $\frac{7}{2}-2\pi^3$

43. If $\frac{7}{2}-2\pi^3$ then $\int_0^2 x^2 f\left(x\right)dx$ equals

(A) 1
(B) $\frac{4}{3}$
(C) $\frac{5}{3}$
(D) $\frac{5}{2}$

44. The value of the integral $\int_{-1}^{1} \sin^{11} x\,dx$ is

(A) $\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}$ (B) $\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}\cdot\frac{\pi}{2}$

(C) 1 (D) 0

45. $\int_{0}^{\pi/2} \frac{dx}{\sin\left(x-\frac{\pi}{3}\right)\cdot\sin\left(x-\frac{\pi}{6}\right)}$ equals

(A) $4\log\sqrt{3}$ (B) $-4\log\sqrt{3}$

(C) $2\log\sqrt{3}$ (D) None of these

46. $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2\,dx,$ where p and q are integers, is equal to

(A) $-\pi$ (B) 0

(C) π (D) 2π

47. $\int_{0}^{1} |\sin 2\pi x|\,dx$ is equal so

(A) 0 (B) $-1/\pi$

(C) $1/\pi$ (D) $2/\pi$

48. If $\int_{0}^{\pi} x f(\sin^3 x + \cos^2 x)dx,\ k\int_{0}^{\pi/2} f(\sin^3 x + \cos^2 x) = dx,$ then k =

(A) $\frac{\pi}{2}$ (B) π

(C) 2π (D) None of these

49. The value of $\int_{0}^{\pi/2} |\sin x - \cos x|\,dx$ is

(A) 0 (B) $2\left(\sqrt{2}-1\right)$

(C) $2\sqrt{2}$ (D) $2\left(\sqrt{2}+1\right)$

50. If f(x) = A. 2^x + B, where f'(1) = 2 and f'(1) = 2 and $\int_0^3 f(x)dx = 7,$ then

(A) $A = \frac{1}{2\log 2}$

(B) $B = \frac{7}{3(\log 2)^2}\left[(\log 2)^2 - 1\right]$

(C) $A = \frac{7}{3(\log 2)^2}\left[(\log 2)^2 - 1\right]$

(D) $B = \frac{1}{\log 2}$

51. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix}$ = 0, if

(A) x, y, z are in AP

(B) x, y, z are in GP

(C) x, y, z are in HP

(D) xy, yz, zx are in AP

52. The parameter, on which the value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$

does not depend upon, is

(A) a

(B) p

(C) d

(D) x

53. If f(x) = $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$, then f (100) is equal to

(A) 0

(B) 1

(C) 100

(D) –100

54. For which value of x will the matrix given below become singular?

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

(A) 4 (B) 6

(C) 8 (D) 12

55. If $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, then A^2 is

(A) idempotent (B) nilpotent

(C) involutory (D) periodic

56. If A is non-scalar, non-identity idempotent matrix of order n $\geq$ 2. Then, minimal polynomial $m_A(x)$ is

(A) x(x –1) (B) x(x + 1)

(C) x(1 –x) (D) $x^2(1 + x)$

57. The number of linearly independent eigenvectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) infinite

58. The square matrix A is defined as

$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$, the diagonal matrix D of A is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

59. The rank of the following (n + 1) x (n + 1) matrix where a is real number

$$\begin{bmatrix} 1 & a & a^2 & \dots & a^n \\ 1 & a & a^2 & \dots & a^n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & a & a^2 & \dots & a^n \end{bmatrix} \text{ is}$$

(A) 1 (B) 2

(C) n (D) Depends on 'a'

60. Multiplication of matrices E and F is G. Matrices E and G are as follows

$$E = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, the value of matrix F is

(A) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} \cos\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

61. The linear operation L(x) is defined by the cross product L(x)= bX_x, where b = $[0\ 1\ 0]^T$ and x = $[X_1\ \ X_2\ \ X_3]^T$ are three dimensional vectors. The 3 × 3 matrix M of this operation satisfies L(x) = M $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Then, the eigen values of M are

(A) 0, + 1, – 1 (B) 1, – 1, 1

(C) i, – i, 1 (D) i, –i, 0

62. Let V be a 3 dimensional vectors space with A and B its subspaces of dimensions 2 and 1, respectively. If $A \cap B = \{0\}$, then

(A) V = A – B (B) V = A + B

(C) V = A . B (D) None of these

63. Dim V, where

$$V = \{a_1, a_2, \ldots\ldots, a_{100} : a_1 + a_2 = 0, a_3 + a_4 = 0\}$$

is

(A) 97 (B) 98

(C) 99 (D) 100

64. Let $f(x) = x^2 - 5x + 6$, and $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then f(A) is equal to

(A) $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -1 & -5 \\ -1 & -1 & 4 \\ -3 & -10 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -1 & 4 \\ -1 & 4 & -10 \\ 4 & -3 & -5 \end{bmatrix}$ (D) None of these

65. $X = [X_1\ X_2\ \ldots.Xn]'$ is an n-tuple non-zero vector. Then n × x matrix V = XX'

(A) has rank zero (B) has rank 1

(C) is orthogonal (D) has rank n

66. Consider a non-homogeneous system of linear equations representing mathematically an over determined system. Such a system will be

(A) consistent having a unique solution

(B) consistent having many solutions

(C) inconsistent having no solution

(D) All of the above

67. Consider the system of equations $A_{(nxn)}x_{(nxt)} = \lambda$ (n × I) where, λ is a scalar. Let (λ_i, x_i) be an eigen pair of an eigen value and its corresponding eigen vector for real matrix A. Let *I* be a (n × n) unit matrix. Which one of the following statements is not correct?

(A) For a homogeneous n×n system of linear equations, $(A - \lambda I) \times = 0$ having a non-trivial solution, the rank of $(A - \lambda I)$ is less than *n*.

(B) For matrix A^m, m being a positive integer, $\left(\lambda_i^m, X_i^m\right)$ will be the eigen pair for all i.

(C) If $A^T = A^{-1}$, then $|\lambda_i| = 1$ for all *i*

(D) If $A^T = A$, then λ_i is real for all i.

68. Eigen values of a matrix S = $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1.

What is the eigen values of the matrix $S^2 = SS$?

(A) 1 and 25 (B) 6 and 4

(C) 5 and 1 (D) 2 and 10

69. Consider the following system of equations

$$2x_1 + x_2 + x_3 = 0$$

$$x_2 - x_3 = 0$$

$$x_1 + x_2 = 0$$

This system has

(A) a unique solution (B) no solution

(C) infinite number of solutions (D) five solutions

70. The system of equations

$$x + y + z = 6$$
$$x + 4y + 6z = 20$$
$$x + 4y + \lambda z = \mu$$

has NO solution for values of λ and μ given by

(A) $\lambda = 6,\ \mu = 20$ (B) $\lambda = 6,\ \mu \neq 20$

(C) $\lambda \neq 6, \mu = 20$ (D) $\lambda \neq 6, \mu \neq 20$

71. For the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ the eigenvalue corresponding to the eigenvector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is

(A) 2 (B) 4

(C) 6 (D) 8

72. Eigenvalues of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigenvalues of the matrix $S^2 = SS$?

(A) 1 and 25 (B) 6 and 4

(C) 5 and 1 (D) 2 and 10

73. The eigen values of a symmetric matrix are all

(A) complex with non-zero positive imaginary fpart

(B) complex with non-zero negative imaginary part

(C) real

(D) pure imaginary

74. The trace and determined of a 2 × 2 matrix are known to be –2 and –35 respectively. It eigenvalues are

(A) –30 and –5 (B) –37 and –1

(C) –7 and 5 (D) 17.5 and –2

75. Find A^{-1} by Cayley–Hamilton theorem, if

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

(A) $\begin{bmatrix} -\frac{1}{5} & \frac{3}{10} \\ \frac{2}{5} & -\frac{1}{10} \end{bmatrix}$ (B) $\begin{bmatrix} -\frac{1}{2} & \frac{5}{10} \\ \frac{2}{5} & -\frac{1}{10} \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{5} & \frac{3}{10} \\ \frac{2}{5} & -\frac{1}{10} \end{bmatrix}$ (D) None of these

76. Consider the second order ordinary differential equation $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ where b and c are real constants if $y = xe^{-5x}$ is solution , then–

(A) both b and c are positive (B) b is positive but c is negative

(C) b is negative but c is positive (D) both b and c are negative

77. Consider the set $V = \{(x_1 - x_2 + x_3, x_1 + x_2 - x_3) : (x_1, x_2, x_3) \in \mathbb{R}^3\}$ Then–

(A) V is not a vector space of $\mathbb{R}^2$

(B) V is a vector subspace of $\mathbb{R}^2$ of dimension 0

(C) V is a vector subspace of $\mathbb{R}^2$ of dimension 1

(D) $V = \mathbb{R}^2$

78. Let A be a 5 × 5 matrix all of whose eigenvalues are zero. Then which of the following statement is always true ?

(A) A = –A (B) $A^t = -A$ (C) $A^t = A$ (D) $A^5 = 0$

79. The radius of convergence of $\sum_{n=0}^{\infty} z^{n!}$ is

(A) 0 (B) 1 (C) 2 (D) ∞

80. The integral $\int_0^2 dx \int_0^{6-x} f(x,y)\,dy$

(A) $\int_0^2\int_0^{y/2} f(x,y)\,dx\,dy + \int_2^6\int_0^{6-y} f(x,y)\,dxdy$ (B) $\int_0^4\int_0^{y/2} f(x,y)\,dx\,dy + \int_4^6\int_0^{6-y} f(x,y)\,dx\,dy$

(C) $\int_0^2\int_0^{6-y} f(x,y)\,dx\,dy + \int_2^4\int_0^{y/2} f(x,y)\,dx\,dy$ (D) $\int_0^4\int_0^{6-y} f(x,y)\,dx\,dy + \int_4^6\int_0^{y/2} f(x,y)\,dx\,dy$

81. The surface area of the solid generated by the revolution of the curve $r^2 = a^2 \cos 3\theta$ about a tangent at the pole is given by–

(A) πa^2 (B) $2\pi a^2$ (C) $4\pi a^2$ (D) None of these

82. Consider the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with real entries suppose it has repeated eigen values pick the correct statement

(A) bc = 0 (B) A is always a diagonal matrix

(C) $\det(A) \geq 0$ (D) det (A) can take any real value

83. Let ϕ be the solution of $y' + iy = x$ such that $\phi(0) = 2$ Then $\phi(\pi)$ equals

(A) $i\pi$ (B) $-i\pi$ (C) π (D) $-\pi$

84. Let f be a solution of the ODE

$$x^2y' + 2xy = 1 \text{ on } 0 < x < \infty$$

Then the limit of f(x) as $x \to \infty$

(A) is zero (B) is one (C) is ∞ (D) does not exist

85. Let $f:[0,\infty) \to [0,\infty)$ satisfy $(f(x))^2 = 1 + 2\int_0^x f(t)\,dt$

Then f(1) is

(A) $\log e^2$ (B) 1 (C) 2 (D) e

86. A subset V of $\mathbb{R}^3$ consisting a vectors (x_1, x_2, x_3) satisfying $x_1^2 + x_2^2 + x_3^2 = k$ is a subspace of $\mathbb{R}^3$ if k is

(A) 0 (B) 1 (C) –1 (D) none of above

87. Let $v_1 = (1, 0)$ $v_2 = (1, -1)$ and $v_3 = (0, 1)$

How many linear transformations $T: R^3 \to R^2$ are there such that $T(v_1) = v_2$, $T(v_2) = v_3$, $T(v_3) = v_1$?

(A) 3! (B) 3 (C) 1 (D) 0

88. Suppose a finite group G has a elements a which is not the identity such that a^{20} is the identity which of the following can not be a possible value for the number of elements of G

(A) 12 (B) 9 (C) 20 (D) 15

89. Let A be a 10 × 10 matrix in which each row has exactly one entry equal to 1 the remaining nine entries of the row being 0 which of the following is not a possible value for the determinant of the matrix A

(A) 0 (B) –1 (C) 10 (D) 1

90. Consider the following statements :

(i) Every principal ideal domain is a Euclidean domain

(ii) The group of units in the ring Z/372 is cyclic

(iii) There is a field with 6^5 elements

(A) (i) is true (ii) and (iii) are false (B) (ii) is true (i) and (iii) are false

(C) (iii) is true (i) and (ii) are false (D) All three statement are false

91. Which of the following statement about the permutation group on {1, 2, 3,....n} is false ?

(A) Every element is a product of transpositions

(B) The element (1,2) (1,3) and (1,2)(3,4) are conjugate

(C) Every element is a product of disjoint cycles

(D) The group is generated by (1,2) and (1,2......n)

92. Consider the two linear maps T_1 and T_2 on V_3 defined as $T(x_1, x_2, x_3) = (0, 0, x_2)$ and $T_2(x_1, x_2, x_3) = (x_1, 0, 0)$ Then

(A) T_1 is idempotent but T_2 is not idempotent

(B) T_2 is idempotent but T_1 is not idempotent

(C) Both T_1, T_2 are idempotent

(D) Neither T_1 nor T_2 are idempotent.

93. If $T : R^4 \to R^3$ be the linear transformation defined by T(x, y, z, w) = (x – y + z + w, x + 2z – w, x + y + 3z – 3w)

Then the dimension of its range is

(A) 3 (B) 2

(C) 1 (D) 0

94. The limit superior and limit inferior of $\left\langle \frac{(-1)^n}{n^2} \right\rangle$ are respectively

(A) –1, –1 (B) 1, 0

(C) 0, 0 (D) 1, 1

95. The sequence <s_n> where $S_n = 1 + \frac{1}{3} + \frac{1}{5} + ... + \frac{1}{2n-1}$ is

(A) convergent (B) monotonically decreasing

(C) not cauchy (D) none of these.

96. Let $f : R^2 \to R$ be defined by

$$f(x,y) = \begin{cases} x^2 + y^2, & \text{if } x \text{ and } y \text{ rational} \\ 0, & \text{otherwise} \end{cases}$$

The–

(A) f is not continuous at (0, 0)

(B) f is continuous at (0, 0) but not differentiable at (0, 0)

(C) f is differentiable only at (0, 0)

(D) f is differentiable everywhere

97. Let y = e^x be a solution of $x\frac{d^2y}{dx^1} - \frac{dy}{dx} + (1-x)y = 0$ Then the second linearly indepen dent solution of this ODE is

(A) xe^{-x} + 1/2 (B) $\frac{1}{2}\left(x - \frac{1}{2}\right)e^{-x}$

(C) $-\frac{1}{2}\left(x + \frac{1}{2}\right)e^{-x}$ (D) $xe^{-x} - \frac{1}{2}$

98. The initial value problem

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0,\ y(0) = 1\left(\frac{dy}{dx}\right)_{x=0} = 0$$

has

(A) A unique solution (B) No solution

(C) Infinitely many solution (D) Two linearly independent solution

99. The area of space bound by first quadrant inside the circle $x^2 + y^2 = 3a^2$ and is bounded by the parabolas

$x^2 = 2ay$ $y^2 = 2x$ (a . 0)

(A) $\frac{\sqrt{2}}{3}a^2$ (B) $\frac{\sqrt{3}}{2}a^2$

(C) $a^2 + \frac{1}{2}$ (D) none of these

100. Which of the following transformations reduce the differential equation $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$ into the form?

$$\frac{dt}{dx} + P(x)(t) = Q(x)$$

(A) t = log z (B) $t = \frac{1}{\log z}$

(C) $t = e^z$ (D) $t = (\log z)^2$

101. With reference to a right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}, \hat{k}$ $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$

If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then

(A) $\vec{\beta}_1 = \frac{3}{2}i + \frac{1}{2}\hat{j}$, $\vec{\beta}_2 = \frac{1}{2}i + \frac{3}{2}\hat{j} - 3\hat{k}$

(B) $\vec{\beta}_1 = \frac{3}{2}i - \frac{1}{2}\hat{j}$, $\vec{\beta}_2 = \frac{1}{2}i + \frac{3}{2}\hat{j} - 3\hat{k}$

(C) $\vec{\beta}_1 = \frac{1}{2}i + \frac{3}{2}\hat{j} - 3\hat{k}$, $\vec{\beta}_2 = \frac{1}{2}i - \frac{3}{2}\hat{j} - 3\hat{k}$

(D) None of these

102. If $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{j} + 4\hat{k}$, then a unit vector along the vector $\vec{a} \times \vec{b}$ is

(A) $\dfrac{-2\hat{j} + \hat{k}}{\sqrt{5}}$ (B) $\dfrac{-\hat{j} - 2\hat{k}}{\sqrt{5}}$

(C) $\dfrac{-2\hat{j} - \hat{k}}{\sqrt{5}}$ (D) None of these

103. A unit vector perpendicular to the plane ABC where A,B, C are the points (3, –1, 2), (1, –1, –3) and (4, –3, 1) respectively, is

(A) $\dfrac{10\hat{k} + 7\hat{j} + 4\hat{k}}{\sqrt{165}}$ (B) $\dfrac{-10\hat{k} + 7\hat{j} + 4\hat{k}}{\sqrt{165}}$

(C) $\dfrac{-10\hat{k} - 7\hat{j} + 4\hat{k}}{\sqrt{165}}$ (D) None of these

104. A vector whose length is 3 and which is perpendicular to each of the vectors

$\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$ is

(A) $\hat{i} + \hat{j} - 2\hat{k}$ (B) $2\hat{i} + 2\hat{j} + \hat{k}$

(C) $2\hat{i} - 2\hat{j} + \hat{k}$ (D) None of these

105. If $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$, then a vector R which satisfies $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R} \cdot \vec{A} = O$, is

(A) $-\hat{i} - 8\hat{j} + 2\hat{k}$ (B) $\hat{i} - 8\hat{j} + 2\hat{k}$

(C) $\hat{i} + 8\hat{j} + 2\hat{k}$ (D) None of these

106. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the points A, B, C then the perpendicular distance from C to the straight line through A and B is

(A) $\dfrac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|}{\left|\vec{b}-\vec{a}\right|}$

(B) $\dfrac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|}{2\left|\vec{b}-\vec{a}\right|}$

(C) $\dfrac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|}{4\left|\vec{b}-\vec{a}\right|}$

(D) None of these

107. Given $\vec{A}=a\hat{i}+b\hat{j}+c\hat{k}, \vec{B}=d\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{C}=2\hat{i}+j-2\hat{k}$. If the vectors $\vec{A}, \vec{B}$ and $\vec{C}$ form a triangle such that $\vec{A}=\vec{B}+\vec{C}$, then

(A) a = –8, b = –4, c = 2, d = –11

(B) a = –8, b = 4, c = –2, d = –11

(C) a = –8, b = 4, c = 2, d = –11

(D) None of these

108. The torque about the point $3\hat{i}-\hat{j}+3\hat{k}$ of a force $4\hat{i}+2\hat{j}+\hat{k}$ through the point $5\hat{i}+2\hat{j}+4\hat{k}$, is

(A) $\hat{i}+2\hat{j}-8\hat{k}$

(B) $\hat{i}+2\hat{j}+8\hat{k}$

(C) $\hat{i}-2\hat{j}-8\hat{k}$

(D) None of these

109. A tetrahedron has vertices at O(0, 0, 0), A = (1, 2, 1), B = (2, 1, 3) and C(–1, 1, 2). Then the angle between the faces OAB and ABC will be

(A) $\cos^{-1}\left(\dfrac{19}{35}\right)$

(B) $\cos^{-1}\left(\dfrac{17}{31}\right)$

(C) 30°

(D) 90°

110. If $\vec{u}=-\hat{i}-2\hat{j}+\hat{k}, \vec{v}=3\hat{i}+\hat{k}$ and $\vec{w}=\hat{j}-\hat{k}$, then the value of $(\vec{u}+\vec{w})\cdot\left[(\vec{u}\times\vec{v})\times(\vec{v}\times\vec{w})\right]$ is

(A) 4

(B) 5

(C) 6

(D) None of these

111. If $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors and mutually orthogonal, then for any vectors $\vec{a}$,

$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) =$

(A) $\vec{O}$ (B) $\vec{a}$

(C) $2\vec{a}$ (D) None of these

112. For any four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$, $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ is equal to

(A) $[\vec{a}\,\vec{b}\,\vec{d}]\vec{c} - [\vec{a}\,\vec{b}\,\vec{c}]\vec{d}$ (B) $[\vec{a}\,\vec{b}\,\vec{c}]\vec{d} - [\vec{a}\,\vec{b}\,\vec{d}]\vec{c}$

(C) $[\vec{a}\,\vec{c}\,\vec{d}]\vec{b} - [\vec{a}\,\vec{b}\,\vec{d}]\vec{c}$ (D) None of these

113. If $\vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a} \cdot \vec{b})\vec{b} = (4 - 2\vec{\beta} - \sin\vec{\alpha})\vec{b} + (\vec{\beta}^2 - 1)\vec{c}$ and $(\vec{c} \cdot \vec{c})\vec{a} = \vec{c}$, while $\vec{b}$ and $\vec{c}$ are non-collinear, then

(A) $\alpha = \frac{\pi}{2}, \beta = -1$ (B) $\alpha = \frac{\pi}{2}, \beta = 1$

(C) $\alpha = \frac{\pi}{3}, \beta = -1$ (D) $\alpha = \frac{\pi}{3}, \beta = 1$

114. If the angle between the vectors (x, 3, –7) and (x, –x, 4) is acute, the interval in which x lies is

(A) (–4, 7) (B) [–4, 7]

(C) R – (–4, 7) (D) R – [–4, 7]

115. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle of 60° to each other, then

(A) $|\vec{a} + \vec{b}| > 1$ (B) $|\vec{a} + \vec{b}| < 1$

(C) $|\vec{a} + \vec{b}| = 1$ (D) None of these

116. The sum of the length of projections of $p\hat{i}+q\hat{j}+r\hat{k}$ on the coordinate axes, where p = 2, q = 3 and r = 1, is

(A) 6 (B) 5

(C) 4 (D) 3

117. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $|\vec{a}|=3,\ |\vec{b}|=4,\ |\vec{c}|=5$ and $\vec{a},\vec{b},\vec{c}$ are pependicular to $\vec{b}+\vec{c},\vec{c}+\vec{a},\ \vec{a}+\vec{b}$ respectively , then $|\vec{a}+\vec{b}+\vec{c}|=$

(A) $6\sqrt{2}$ (B) $4\sqrt{2}$

(C) $3\sqrt{2}$ (D) $5\sqrt{2}$

118. If $\vec{a}\cdot\vec{b}=\beta$ and $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}$ is equal to

(A) $(\beta\vec{a}-\vec{a}\times\vec{c})/\vec{a}^2$ (B) $(\beta\vec{a}+\vec{a}\times\vec{c})/\vec{a}^2$

(C) $(\beta\vec{c}-\vec{a}\times\vec{c})/\vec{a}^2$ (D) $(\beta\vec{c}+\vec{a}\times\vec{c})/\vec{a}^2$

119. If $\vec{a}=(2,1,1),\ \vec{b}=(1,0,3),\ \vec{c}=(2,1,3)$ and $\vec{a}\times(\vec{b}\times\vec{c})=x\vec{a}+y\vec{b}+z\vec{c}$, then (x, y, z) =

(A) (0, –8, 5) (B) (8, 0, –5)

(C) (0, 8, –5) (D) (8, –5, 0)

120. If $\hat{i},\hat{j},\hat{k}$ is orthogonal system of vectors, $\vec{a}$ is a vector and $\vec{a}\times\hat{i}+2\vec{a}-5\hat{j}=0$ then $\vec{a}=$

(A) $2\hat{j}+\hat{k}$ (B) $2\hat{j}-\hat{k}$

(C) $2\hat{i}+\hat{j}$ (D) $2\hat{j}+\hat{j}$

Ques	Ans	Ques	Ans	Ques	Ans	Ques	Ans	Ques	Ans	Ques	Ans
1	D	21	B	41	B	61	D	81	C	101	B
2	A	22	B	42	A	62	B	82	C	102	C
3	D	23	B	43	C	63	B	83	B	103	C
4	D	24	C	44	D	64	A	84	A	104	C
5	A	25	B	45	B	65	B	85	B	105	A
6	B	26	B	46	D	66	D	86	B	106	A
7	A	27	C	47	D	67	B	87	B	107	C
8	A	28	D	48	B	68	A	88	B	108	A
9	C	29	C	49	B	69	C	89	C	109	A
10	B	30	C	50	B	70	B	90	A	110	C
11	B	31	B	51	B	71	C	91	B	111	C
12	C	32	A	52	B	72	A	92	B	112	A
13	B	33	A	53	A	73	C	93	A	113	B
14	A	34	B	54	A	74	C	94	C	114	C
15	A	35	B	55	C	75	A	95	C	115	A
16	B	36	C	56	A	76	C	96	C	116	A
17	C	37	D	57	B	77	D	97	C	117	D
18	C	38	A	58	C	78	D	98	A	118	A
19	D	39	B	59	A	79	B	99	D	119	C
20	A	40	A	60	C	80	B	100	B	120	A

HINTS AND SOLUTIONS

1.(D) Since $f(x) = (x + 2)e^{-x}$

$\therefore \quad f'(x) = (x + 2)(-e^{-x}) + e^{-x}.1$

$f'(x) = -e^{-x}(x+1)$

If f(x) is decreasing function, then $f'(x) < 0$

$\Rightarrow \quad -e^{-x}(x + 1) < 0$

or $\quad x + 1 > 0$

$\therefore \quad x \in (-1, \infty)$

and if f(x) is increasing function,then f'(x)>0

$\Rightarrow \quad -e^{-x}(x+1) > 0$

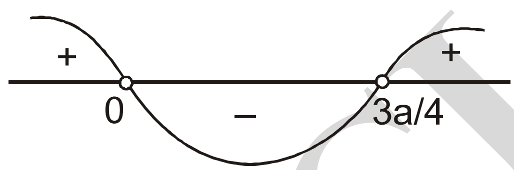
or $\quad x + 1 < 0$

$\therefore \quad x \in (-\infty, -1)$

2.(A) Given, f(x) = $x\sqrt{(ax - x^2)}, a > 0$

$$\therefore \quad f'(x) = x\frac{1}{2\sqrt{(ax - x^2)}}.(a - 2x) + \sqrt{(ax - x^2)}.1$$

$$= \frac{(3ax - 4x^2)}{2\sqrt{(ax - x^2)}} = \frac{-4x(x - 3a/4)}{2\sqrt{(ax - x^2)}}$$



For f'(x)> 0(Increasing)

then, $x \in (0, 3a/4)$

and for f'(x) < 0 (decreasing)

Then, $x \in (-\infty, 0) \cup (3a/4, \infty)$

3.(D) $g'(x) = f'(\tan^2 x - 2\tan x + 4).(2\tan x - 2), \sec^2 x$ ....(i)

$\therefore$ f''(x) > 0 $\Rightarrow$ f'(x) is increasing function.

So f' $(\tan^2 x - 2\tan x + 4) = f'(\tan x - 1)^2 + 3) > f'(3) = 0$

$$\forall x \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

Also, (tan x – 1) > 0, for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

So, g(x) is increasing function in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

4.(D) $\therefore$ $$f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$$

$\therefore$ $$f'(x) = \frac{8x}{\left(e^{x^2} + e^{-x^2}\right)^2}$$

$$= \begin{cases} > 0, & x > 0 \\ < 0, & x < 0 \\ 0, & x = 0 \end{cases}$$

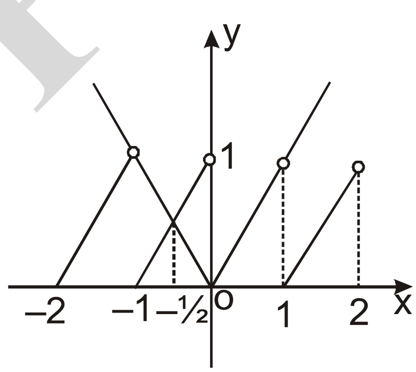
5.(A) $\therefore$ $f'(x) = |x| - \{x\}$

$\therefore$ f(x) is decreasing

$\therefore$ f'(x) < 0

$\Rightarrow$ $|x| - \{x\} < 0$

$\Rightarrow$ $|x| < \{x\}$



6.(B) $f(x) = (x^{a-b})^{a+b} \cdot (x^{b-c})^{b+c}.(x^{c-a})^{c+a}$

$$= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2-a^2} = x^0 = 1.$$

$\therefore$ f'(x) = 0.

7.(A) We have,

$f(x) = (\sin^{m-n}x)^{m+n}\cdot(\sin^{n-p}x)^{n+p}.(\sin^{p-m}x)^{p+m}$

$$= \sin^{m^2-n^2} x \cdot \sin^{n^2-p^2} x \cdot \sin^{p^2-m^2} x$$

$$= (\sin x)^{m^2-n^2+n^2-p^2-m^2} = (\sin x)^{\circ} = 1$$

$\therefore \quad f'(x) = 0$

8.(A) We have, $y = \left(x + \sqrt{1+x^2}\right)^n$ ... (1)

Differentiating Eq. (1), we get

$$\frac{dy}{dx} = n\left[x + \sqrt{1+x^2}\right]^{n-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{n\left[x + \sqrt{1+x^2}\right]}{\sqrt{1+x^2}}$$

or $\dfrac{dy}{dx} = \dfrac{ny}{\sqrt{1+x^2}} \Rightarrow y_1^2\left(1+x^2\right) = n^2y^2.$

Again differentiating, we get

$2y_1y_2(1 + x^2) + 2xy_1^2 = 2n^2yy_1$

Dividing by $2y_1$, we get

$y_2(1 + x^2) + xy_1 = n^2y$

or $$\frac{d^2y}{dx^2}\left(1+x^2\right)+x\frac{dy}{dx}=n^2y.$$

9.(C) We have,

$$\phi(x)=\log_5 \log_3 x=\log_5\left(\frac{\log x}{\log 3}\right)$$

$$= \log_5 (\log x) – \log_5 (\log 3)$$

$$=\frac{\log(\log x)}{\log 5}=\log_5(\log 3)$$

$$\phi'(x)=\frac{1}{\log 5}\cdot\frac{1}{\log x}\cdot\frac{1}{x}-0$$

$\therefore$ $$\phi'(e)=\frac{1}{\log 5}\cdot\frac{1}{\log e}\cdot\frac{1}{e}$$

$$=\frac{1}{e\log 5}.$$

10.(B) We have, $y=f\left(\frac{2x-1}{x^2+1}\right)$

$$\Rightarrow \frac{dy}{dx}=f'\left(\frac{2x-1}{x^2+1}\right)\cdot\left[\frac{\left(x^2+1\right)2-(2x-1)\cdot 2x}{\left(x^2+1\right)^2}\right]$$

$$=\sin\left(\frac{2x-1}{x^2+1}\right)^2\cdot\left[\frac{2+2x-2x^2}{\left(x^2+1\right)^2}\right]$$

$$\left[\because f'(x) = \sin x^2, \therefore f'\left(\frac{2x-1}{x^2+1}\right) = \sin\left(\frac{2x-1}{x^2+1}\right)^2\right]$$

11.(B) Given f(x) = $\log_x(\log x)$

$$= \log_e(\log_e x) \times \log_x e = \frac{\log_e(\log_e x)}{\log_e x}$$

$$\therefore \qquad f'(x) = \frac{\log_e x \times \frac{1}{\log_e x} \times \frac{1}{x} - \log_e(\log_e x) \times \frac{1}{x}}{(\log_e x)^2}$$

$$= \frac{\frac{1}{x}\left[1 - \log_e(\log_e x)\right]}{\left[\log_e x\right]^2}$$

$$\therefore \qquad f'(x) \text{at } x = e = \frac{\frac{1}{e}\left[1 - \log_e(\log_e e)\right]}{\left[\log e\right]^2} = \frac{1}{e}$$

$[\because \log_e e = 1 \text{and} \log_e 1 = 0]$

12.(C) $\therefore$ f(x) = x^x

$\therefore$ $f'(x) = x^x (1 + \ln x) < 0$ (given)

$\therefore$ $x^x > 0$

$\therefore$ $1 + \ln x < 0$

$\Rightarrow$ $\ln x < -1$

$\Rightarrow$ $x < e^{-1}$

$\therefore$ $x \in (0, 1/e)$

13.(B) We have,

$$y=\sum_{r=1}^{x}\tan^{-1}\frac{1}{1+r+r^2}=\sum_{r=1}^{x}\tan^{-1}\left(\frac{(r+1)-r}{1+(r+1)r}\right)$$

$$=\sum_{r=1}^{x}\left[\tan^{-1}(1+1)-\tan^{-1}r\right]$$

= [$\tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + ... + \tan^{-1}x - \tan^{-1}(x-1) + \tan^{-1}(x+1)$ $- \tan^{-1}x$]

= [$\tan^{-1}(x+1) - \tan^{-1}$]

$$\therefore \qquad \frac{dy}{dx}=\frac{1}{1+(x+1)^2}.$$

14.(A) We have,

$x = t^t = e^{t\log t}$

$$\Rightarrow \qquad \frac{dx}{dt}=e^{t\log t}(1=\log t)=t^t(1+\log t)$$

Also, $y=t^{t^t} \Rightarrow \log y = t^t \log t = e^{t\log t}\cdot \log t$

$$\Rightarrow \qquad \frac{1}{y}\frac{dy}{dt}=e^{t\log t}(1+\log t)\log t+e^{t\log t}\cdot\frac{1}{t}$$

$$\Rightarrow \qquad \frac{dy}{dt}=t^{t^t}\cdot t^t\left[(1+\log t)\log t+\frac{1}{t}\right]$$

$$\therefore \qquad \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{t^{t^t}\left[(1+\log t)\log t+\frac{1}{t}\right]}{(1+\log t)}.$$

15.(A) Let y = f(tan x) and u = g(sec x)

$$\Rightarrow \quad \frac{dy}{dx} = f'(\tan x) = \sec^2 x$$

$$\text{and} \quad \frac{du}{dx} = g'(\sec x)\cdot \sec x \tan x$$

$$\therefore \quad \frac{dy}{du} = \frac{dy}{dx}\bigg/\frac{du}{dx} = \frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x \tan x}$$

$$\therefore \quad \left.\frac{dy}{du}\right]_{x=\frac{\pi}{4}} = \frac{f'\left(\tan\frac{\pi}{4}\right)}{g'\left(\sec\frac{\pi}{4}\right)\sin\frac{\pi}{4}}$$

$$= \frac{f'(1)}{g'(\sqrt{2})\cdot\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}\times 2}{4} = \frac{1}{\sqrt{2}}.$$

16.(B) We have,

$$\frac{d}{dx}\left[\frac{(1+x^2+x^4)(1-x^2+x^4)}{(1+x^2+x^4)}\right] = ax^3 + bx$$

$$\Rightarrow \quad \frac{d}{dx}(1-x^2+x^4) = ax^3 + bx$$

$\Rightarrow \quad -2x + 4x^3 = ax^3 + bx \Rightarrow a = 4$ and $b = -2$.

17.(C) $f'(x) = -2\cos x\sin x - 2\cos\left(x+\frac{\pi}{3}\right)\sin\left(x+\frac{\pi}{3}\right)$

$$+\cos x\sin\left(x+\frac{\pi}{3}\right)+\sin x\cos\left(x+\frac{\pi}{3}\right)$$

$$=-\sin 2x-\sin\left(2x+\frac{2\pi}{3}\right)+\sin\left(x+x+\frac{\pi}{3}\right)$$

$$=-2\sin\left(2x+\frac{\pi}{3}\right)\cos\frac{\pi}{3}+\sin\left(2x+\frac{\pi}{3}\right)$$

$$=-2\sin\left(2x+\frac{\pi}{3}\right)+\sin\left(2x+\frac{\pi}{3}\right)=0.$$

$\Rightarrow$ f(x) = constant for all x.

But, $f(0) = \cos^2 0 + \cos^2 \frac{\pi}{3}+\sin 0\cdot\sin\frac{\pi}{3}=\frac{5}{4}$

$\therefore$ $f(x)=\frac{5}{4}$ for all x.

Thus, $(gof)(x)=g\left[f(x)\right]=\left(\frac{5}{4}\right)=3.$

18.(C) f(x) is a polynomial of degree 90. f'(x) reduces the degree of f(x) by one. Thus, in order to get a polynomial of degree 20, we must reduce the degree of f(x) by 70. Hence,f(x) should be differentiated 70 times to get a polynomial of degree 20.

$\therefore$ n = 70.

19.(D) We have,

$f'(x)= 6x + 4g'(1)$ ... (1)

$f''(x) = 6$ ... (2)

$g'(x) = 4x + 3f'(2)$ ... (3)

$g''(x) = 4$ ... (4)

From (1),

$f'(1) = 6 + 4g'(1) = 6 + 4\,[4 + 3f'(2)]$

$[\because\ g'(1) = 4 + 3f'(2)] = 22 + 12f'(2)$

From (3),

$g'(3) = 8 + 3f'(2) = 8 + 3\,[12 + 4g'(1)]$

$[\because\ f'(2) = 12 + 4g'(1)] = 44 + 12\,g'(1)$

Also, from (2) and (4), $f'''(3) + g''(2) = 6 + 4 = 10$.

20.(A) We have,

$$y_1 = (n-1)x^{n-2}\log x + x^{n-1}\cdot\frac{1}{x}$$

$\Rightarrow \quad xy_1 = (n-1)x^{n-1}\log x + x^{n-1}$

$= (n-1)y + x^{n-1}$ .... (1)

Differentiating again w.r.t. x, we get

$y_1 + xy_2 = (n-1)\,y_1 + (n-1)x^{n-2}$

$\Rightarrow \quad x^2y_2 + xy_1 = (n-1)\,xy_1 + (n-1)x^{n-1}$

$= (n-1)\,xy_1 + (n-1)\,(xy_1 - (n-1)y)$ [Using (1)]

$\Rightarrow \quad x^2y_2 + (3-2n)xy_1 + (n-1)^2\,y = 0.$

21.(B) We have,

$$4x^2 = (y^{1/5} + y^{-1/5})^2 = y^{2/5} + y^{-2/5} + 2$$

$= (y^{1/5} - y^{-1/5})^2 + 4$

$\Rightarrow \quad y^{1/5} - y^{-1/5} = 2\sqrt{x^2-1}.$

Adding with the given relation, we get

$$2y^{1/5} = 2\left[x+\sqrt{x^2-1}\right] \Rightarrow y = \left[x+\sqrt{x^2-1}\right]^5$$

$$\Rightarrow \quad y_1 = 5\left(x+\sqrt{x^2-1}\right)^4\left(1+\frac{x}{\sqrt{x^2-1}}\right)$$

$$\Rightarrow \quad y_1 = \frac{5\left(x+\sqrt{x^2-1}\right)^5}{\sqrt{x^2-1}} = \frac{5y}{\sqrt{x^2-1}}$$

$$\Rightarrow \quad \left(x^2-1\right)y_1^{\,2} = 25y^2$$

Differentiating again w.r.t. x, we get

$(x^2 - 1)\, 2y_1y_2 + y_1^{\,2} \cdot 2x = 50yy_1$ [Dividing by $2y_1$]

$\Rightarrow \quad (x^2 - 1)\, y_2 + xy_1 = 25\, y. \quad \therefore \quad k = 25.$

22.(B) We have, $\log y = \tan^2 x \cdot \log \tan x$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = 2\tan x \sec^2 x \cdot \log\tan x + \tan^2 x \cdot \frac{1}{\tan x}\sec^2 x$$

$$\Rightarrow \quad \frac{dy}{dx}\ [(\tan x)^{\tan x}]^{\tan x} \cdot \tan x \sec^2 x\,(2 \log \tan x + 1)$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right]_{x=\frac{\pi}{4}} = 1\cdot1\cdot2\cdot(2\cdot0 + 1) = 2.$$

23.(B) We have,

$$y = (1 + x)(1 + x^2)(1 + x^4) \ldots \left(1+x^{2^n}\right)$$

$$= \frac{(1-x)(1+x)\left(1+x^2\right)\left(1+x^4\right)\ldots\left(1+x^{2^n}\right)}{1-x} = \frac{1-x^{2^{n+1}}}{1-x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(1-x)\cdot -2^{n+1}\cdot x^{2n+1} + \left(1-x^{2^{n+1}}\right)}{(1-x)^2}$$

$$\therefore \quad \left.\frac{dy}{dx}\right]_{x=0} = 1.$$

24.(C) Since, $f(x) = \dfrac{2\sin x\cos x\cos 2x\cos 4\cos 8x}{2\sin x} = \dfrac{\sin 16x}{2^4 \sin x}$

$$\Rightarrow \quad f'(x) = \frac{1}{16}\left[\frac{\sin x \cdot \cos 16x \cdot 16 - \sin 16x \cdot \cos x}{\sin^2 x}\right]$$

$$\therefore \quad f'\left(\frac{\pi}{4}\right) = \frac{1}{16}\left[\frac{\frac{1}{\sqrt{2}} \cdot 1 \cdot 16 - \frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)}\right] = \sqrt{2}.$$

25.(B) Since, $1 < x < 2$. $\quad \therefore$ [x] = 1

$$\therefore \quad f(x) = \sin\left(\frac{\pi}{2} - x^5\right) = \cos x^5$$

$$\Rightarrow \quad f'(x) = -\sin x^5 \cdot 5x^4$$

$$\Rightarrow \quad f'\left(\sqrt[5]{\frac{\pi}{2}}\right) = -5\left(\frac{\pi}{2}\right)^{4/5} \cdot \sin\frac{\pi}{2} = -5\left(\frac{\pi}{2}\right)^{4/5}.$$

26.(B) $\displaystyle\int_3^5 \frac{x^2}{x^2-4}dx = \int_3^5 \frac{(x^2-4)+4}{x^2-4}dx = \int_3^5 \left(1 + \frac{4}{x^2-4}\right)_{dx}$

$$= \left[x + 4 \cdot \frac{1}{2 \cdot 2}\log\left|\frac{x-2}{x+2}\right|\right]_0^5$$

$$= (5-3) + \log\frac{3}{7} - \log\frac{1}{5}$$

$$= 2 + \log_e\left(\frac{15}{7}\right).$$

27.(C) $\left|\int_{10}^{19} \frac{\sin x\, dx}{1+x^8}\right| \le \int_{10}^{19} \left|\frac{\sin x}{1+x^8}\right| dx$

$$\le \int_{10}^{19} \frac{1}{1+x^8} dx \le \int_{10}^{19} \frac{dx}{x^8}$$

$$= \frac{-1}{7}\left[x^{-7}\right]_{10}^{19} = \frac{1}{7}10^{-7} - \frac{1}{7}19^{-7} < \frac{1}{7}10^{-7} < 10^{-7}$$

28.(D) Let $I = \int_a^b \frac{|x|}{x} dx$

(i) If $0 \le a < b$, then |x| = x,

$\therefore \quad I = \int_a^b \frac{x}{x} dx = \int_a^b 1 dx = b - a = |b| - |a|.$

(ii) If $a < b \le 0$, then |x| = –x,

$\therefore \quad I = \int_a^b \frac{-x}{x} dx = a - b = -b - (-a) = |b| - |a|.$

(iii) If a < 0 < b, then

$$I = \int_a^0 \frac{|x|}{x} dx + \int_0^b \frac{|x|}{x} dx = \int_a^0 \frac{-x}{x} dx + \int_0^b \frac{x}{x} dx$$

$$= -\int_a^0 dx + \int_0^b dx = a + b = b - (-a) = |b| - |a|.$$

Hence (D) is correct answer.

29.(C) f(0) = a, f(1) = a + b + c, $f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}.$

$\therefore \quad I = \int_0^1 f(x) dx = \int_0^1 \left(a + bx + cx^2\right) dx$

$$=\left[ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right]_0^1.$$

$$=\frac{1}{6}(6a+3b+2c)$$

$$=\frac{1}{6}\left[f(0)+4f\left(\frac{1}{2}\right)+f(1)\right].$$

30.(C) Since $1-x\geq 0$ when $-1\leq x\leq 1.$

$$\therefore \qquad \int_{-1}^{1}|(1-x)|dx=\int_{-1}^{1}(1-x)dx$$

$$\left(x-\frac{x^2}{2}\right)\Bigg|_{-1}^{1}=2.$$

31.(B) Let $I=\int_0^1\frac{dx}{2e^x-1}=\int_0^1\left(\frac{2e^x}{2e^x-1}-1\right)dx$

$$=\left[\log\left(2e^x-1\right)-x\right]_0^1=\log(2e-1)-1.$$

$\therefore \qquad$ p = 1, q = 2, r = 1.

32.(A) Since $0\leq x\leq 1 \quad \Rightarrow \quad 1\leq 1+x^4\leq 2$

$$\Rightarrow \quad 1\leq\sqrt{1+x^4}\leq\sqrt{2} \quad \Rightarrow \quad \frac{1}{\sqrt{2}}\leq\frac{1}{\sqrt{1+x^4}}\leq 1$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\leq\int_0^1\frac{dx}{\sqrt{1+x^4}}\leq 1.$$

Hence $\left[\frac{1}{\sqrt{2}},1\right]$ is the smallest interval such that

$$\int_0^1\frac{dx}{\sqrt{1+x^4}}\in\left[\frac{1}{\sqrt{2}},1\right].$$

33.(A) We have, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x+0)}{h}$

$$= \lim_{h\to 0} \frac{f(x)\cdot f(h) - f(x)\cdot f(0)}{h}$$

$$= f(x)\cdot \lim_{h\to 0} \frac{f(h)-f(0)}{h} = f(x)\cdot f'(0) = k\cdot f(x)$$

$\Rightarrow \quad \frac{d}{dx} f(x) = k\ f(x) \Rightarrow \frac{df(x)}{f(x)} = k\ dx$

$\Rightarrow \quad$ log f(x) = kx + c. $\therefore$ f((x) = e^{kx+c} = ae^{kx}.

34.(B) $\int_0^{\frac{\pi}{6}} \cos^4 3\theta \sin^3 6\theta\, d\theta = 8 \int_0^{\frac{\pi}{6}} \cos^6 3\theta \sin^3 3\theta \cos 3\theta\, d\theta$

Put $\quad \sin 3\theta = t$

$\Rightarrow \quad 3\cos 3\theta\, d\theta = dt$

$\Rightarrow \quad I = \frac{8}{3}\int_0^1 \left(1-t^2\right)^3 t^3.d$

$$= \frac{8}{3}\int_0^1 \left(t^6 - 2t^2 + 3t^4\right) t^3\, dt = \frac{8}{3}\int_0^1 \left(t^3 - t^9 - 3t^5 + 3t^7\right) dt$$

$$= \frac{8}{3}\left[\frac{t^4}{4} - \frac{t^{10}}{10} - \frac{3t^6}{6} + \frac{3t^8}{8}\right]_0^1 = \frac{8}{3}\left[\frac{1}{4} - \frac{1}{10} - \frac{3}{6} + \frac{3}{8}\right]$$

$$= \frac{8}{3}\left[\frac{30-12-60+45}{120}\right] = \frac{1}{15}$$

35.(B) Let $I = \int_0^1 \frac{\tan^{-1} x}{x} dx = \int_0^{\pi/2} \frac{\frac{z}{2}\left(\frac{1}{2}\sec^2\frac{z}{2}\right)}{\tan\frac{z}{2}} dz$

$$\left[\begin{array}{l}\text{Putting } \tan^{-1}x = \dfrac{z}{2} \\ \text{i.e. } x = \tan\dfrac{z}{2} \Rightarrow dx = \dfrac{1}{2}\sec^2\dfrac{z}{2}dz\end{array}\right]$$

$$= \frac{1}{2}\int_0^{\pi/2}\frac{z}{\sin z}dz$$

$$= \frac{1}{2}\int_0^{\pi/2}\frac{x}{\sin x}dx.$$

36.(C) $\displaystyle\int_0^{\pi/3}\frac{\cos x}{3+4\sin x}dx = \frac{1}{4}\int_3^{3+2\sqrt{3}}\frac{dt}{t} = \frac{1}{4}\left[\log t\right]_3^{3+2\sqrt{3}}$

$$\left[\text{Putting } 3+4\sin x = t \Rightarrow \cos x\,dx = \frac{1}{4}dt\right]$$

$$= \frac{1}{4}\left[\log\left(3+2\sqrt{3}\right)-\log 3\right] = \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right).$$

$\therefore \qquad k = \dfrac{1}{4}.$

37.(D) Let $\displaystyle I = \int_0^1\left(1+e^{-x^2}\right)dx = \int_0^1 1dx + \int_0^1 e^{-x^2}dx$

$$= 1 + \int_0^1 e^{-x^2}dx$$

Let $f(x) = e^{-x^2}$. Then, f(x) decreases in 0 < x < 1,

$\therefore$ f(1) < f(x) < f(0)

$\Rightarrow \qquad e^{-1} < e^{-x^2} < e^0 \Rightarrow \displaystyle\frac{1}{e}\int_0^1 dx < \int_0^1 e^{-x^2}dx < \int_0^1 dx$

$$\Rightarrow \quad 1+\frac{1}{e}<1+\int_e^1 e^{x^2}dx<2$$

$\therefore \quad 1 + e^{-1} < I < 2.$

38.(A) We have, $f(x) = ae^{2x} + be^{x} + cx$

$\Rightarrow \quad f'(x) = 2ae^{2x} + be^{x} + c.$

$f(0) = -1 \Rightarrow a + b = -1$ ... (1)

$f'(\log 2) = 31 \Rightarrow 2ae^{2\log 2} + be^{\log 2} + c = 31$

$\Rightarrow \quad 8a + 2b + c = 31$ ... (2)

Also, $\int_0^{\log 4}\left[f(x)-cx\right]dx=\frac{39}{2}\Rightarrow\int_0^{\log 4}\left(ae^{2x}-be^{x}\right)dx$

$=\frac{39}{2}$

$$\Rightarrow \quad \left[\frac{a}{2}e^{2x}+be^{x}\right]_0^{\log 4}=\frac{39}{2}$$

$$\Rightarrow \quad \frac{a}{2}e^{2\log 4}+be^{\log 4}-\left(\frac{a}{2}+b\right)=\frac{39}{2}$$

$$\Rightarrow \quad \frac{1}{2}(15a+6b)=\frac{39}{2}$$

$\Rightarrow \quad 15a + 6b = 39$ ... (3)

Solving (1), (2) and (3), we get

$a = 5$, $b = -6$ and $c = 3$.

39.(B) Let $f(x)=\int_0^x\left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx.$

$\Rightarrow \quad f'(x) = (1 + \cos^8 x)(ax^2 + bx + c).$

Clearly, f(x) is continuous on [1, 2] and derivable on (1, 2).

Also f(1) = f(2) (Given).

$\therefore$ By Rolle's theorem, there exists a point $t \in (1, 2)$ such that $f'(t) = 0$

$\Rightarrow$ $(1 + \cos^8 t)(at^2 + bt + c) = 0$

$\Rightarrow$ $at^2 + bt + c = 0$ $(\because 1 + \cos^8 t \neq 0)$

$\Rightarrow$ the quadratic equation $ax^2 + bx + C = 0$ has atleast one root in (1, 2).

40.(A) Let $f(x) = \cos x \cdot \log\frac{1+x}{1-x}$

$\Rightarrow$ $f(-x) = \cos(-x)\cdot\log\frac{1+x}{1-x}$

$= -\cos x \log\frac{1+x}{1-x} = -f(x)$

$\Rightarrow$ f(x) is an odd function.

$\therefore$ $\int_{-1/2}^{1/2} \cos x \log\frac{1+x}{1-x}\,dx = 0.$

41.(B) Let $f(x) = \sin^{10}x\,(6x^9 - 25x^7 + 4x^3 - 2x)$

$\Rightarrow$ $f(-x) = \sin 10x\,(-6x^9 + 25x^7 - 4x^3 + 2x) = -f(x)$

$\Rightarrow$ f(x) is an odd function.

$\therefore$ $\int_{-\pi/2}^{\pi/2} \sin^{10}x\left(6x^9 - 25x^7 + 4x^3 - 2x\right)dx = 0.$

42.(A) Let $f(x) = (1 - x^2)\sin x \cos^2 x$

$\Rightarrow$ $f(-x) = (1 - x^2)\sin(-x)\cos^2(-x)$

$= -(1 - x^2)\sin x \cos^2 x = -f(x)$

$\Rightarrow$ f(x) is an odd function.

$\therefore$ $\int_{-\pi}^{\pi} \left(1 - x^2\right)\sin x \cos^2 x\,dx = 0.$

43.(C) $\int_0^2 x^2 f(x)\,dx = \int_0^1 x^2 f(x)\,dx + \int_1^2 x^2 f(x)\,dx$

$= \int_0^1 x^3\,dx + \int_0^2 x^2(x-1)\,dx$

$$= \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{4} + \left[\left(\frac{16}{4} - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right] = \frac{5}{3}.$$

44.(D) Let f(x) = $\sin^{11}x \Rightarrow f(-x) = \sin^{11}(-x)$

$= -\sin^{11}x = -f(x).$

$\Rightarrow$ f(x) is an odd function

$$\therefore \quad \int_{-1}^{1} \sin^{11} x\, dx = 0.$$

45.(B) Let $I = \displaystyle\int_0^{\pi/2} \frac{dx}{\sin\left(x - \frac{\pi}{3}\right) \cdot \sin\left(x - \frac{\pi}{6}\right)}$

$$= 2\int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right) dx}{\sin\left(x - \frac{\pi}{3}\right) \cdot \sin\left(x - \frac{\pi}{6}\right)}$$

$$= 2\int_0^{\pi/2} \frac{\sin\left[\left(x - \frac{\pi}{6}\right) - \left(x - \frac{\pi}{3}\right)\right]}{\sin\left(x - \frac{\pi}{3}\right) \cdot \sin\left(x - \frac{\pi}{6}\right)} dx$$

$$= 2\int_0^{\pi/2} \left[\cot\left(x - \frac{\pi}{3}\right) - \cot\left(x - \frac{\pi}{6}\right)\right] dx$$

$$= 2\left[\log \frac{\sin\left(x - \frac{\pi}{3}\right)}{\sin\left(x - \frac{\pi}{6}\right)}\right]_0^{\pi/2} = -4\log\sqrt{3}.$$

46.(D) Let $I = \int_{-\pi}^{\pi} \left(\cos px - \sin qx\right)^2 dx$

$$= \int_{-\pi}^{\pi} \left(\cos^2 px + \sin^2 qx - 2\sin qx \cos px\right) dx$$

$$= \int_{-\pi}^{\pi} \left(\cos^2 px + \sin^2 qx\right) dx - 1\int_{-\pi}^{\pi} \sin qx \cos px \, dx$$

$$= 2\int_{0}^{\pi} \left(\cos^2 px + \sin^2 qx\right) - 0$$

[$\because$ sin qx cos px is odd function]

$$= \int_{0}^{\pi} \left(2 + \cos 2px - \cos 2qx\right) dx = 2\pi.$$

47.(D) Since sin 2px is positive for $0 < x \le \frac{1}{2}$ and negative for $\frac{1}{2} < x < 1$.

$$\therefore \qquad \left|\sin 2\pi x\right| = \begin{bmatrix} \sin 2\pi x, \text{for } 0 < x \le \frac{1}{2} \\ -\sin 2\pi x, \text{ for } \frac{1}{2} < x < 1 \end{bmatrix}$$

$$\therefore \qquad \int_{0}^{1} \left|\sin 2\pi x\right| dx = \int_{0}^{1} \sin 2\pi x \, dx + \int_{1/2}^{1} \left(-\sin 2\pi x\right) dx$$

$$= \left[\frac{-\cos 2\pi x}{2\pi}\right]_{0}^{1/2} + \left[\frac{\cos 2\pi x}{2\pi}\right]_{1/2}^{1} = \frac{2}{\pi}.$$

48.(B) Let $I = \int_{0}^{\pi} x f\left(\sin^3 x + \cos^2 x\right) dx$

$$= \int_{0}^{\pi} (\pi - x) f\left(\sin^3(\pi - x) + \cos^2(\pi - x)\right) dx$$

$$= \pi\int_0^{\pi} f\left(\sin^3 x + \cos^2 x\right)dx - I$$

$$\Rightarrow \qquad I = \frac{\pi}{2}\int_0^{\pi} f\left(\sin^3 x + \cos^2 x\right)dx$$

$$= \frac{\pi}{2}\cdot 2\int_0^{\pi/2} f\left(\sin^3 x + \cos^2 x\right)dx$$

$$= \pi\int_0^{\pi/2} f\left(\sin^3 x + \cos^2 x\right)dx \quad \therefore \qquad k = \pi.$$

49.(B) $\int_0^{\pi/2} |\sin x - \cos x|dx$

$$= \int_0^{\pi/4} |\sin x - \cos x|dx + \int_{\pi/4}^{\pi/2} |\sin x - \cos x|dx$$

$$= -\int_0^{\pi/4} (\sin x - \cos x)dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)dx$$

$$= [\cos x + \sin x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right) + \left(-1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \left(2\sqrt{2} - 2\right).$$

50.(B) We have f(x) = A.2^x + B

$\Rightarrow$ f'(x) = A · 2^x log 2

f'(1) = 2 $\Rightarrow$ 2 = 2A log 2 $\Rightarrow$ $A = \frac{1}{\log 2}$.

$$\int_0^3 f(x)dx = 7 \Rightarrow \int_0^3 \left(\frac{1}{\log 2}\cdot 2^x + B\right)dx = 7$$

$$\Rightarrow \quad \left[\frac{1}{(\log 2)^2}\cdot 2^x + Bx\right]_0^3 = 7$$

$$\Rightarrow \quad \frac{8}{(\log 2)^2} + 3B - \frac{1}{(\log)^2} = 7$$

$$\Rightarrow \quad B = \frac{7}{3(\log 2)^2}\left[(\log 2)^2 - 1\right]$$

$$\therefore \quad A = \frac{1}{\log 2} \text{ and } B = \frac{7}{3(\log 2)^2}\left[(\log 2)^2 - 1\right].$$

51.(B) Given, $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$

Applying $C_1 \to C_1 - (p\,C_2 + C_3)$

$$\Rightarrow \quad \begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -(xp^2+yp+yp+z) & xp+y & yp+z \end{vmatrix}$$

$\Rightarrow \quad -(xp^2 + 2yp + z)(xz - y^2) = 0$

$\therefore \quad$ Either $xp^2 + 2yp + z = 0$ or $y^2 = xz$

$\Rightarrow \quad$ x, y, z are in GP.

52.(B) Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$

Applying $C_1 \to C_1 + C_3$

$$\Rightarrow \qquad \Delta = \begin{vmatrix} 1+a^2 & a & a^2 \\ \cos(p-d)x+\cos(p+d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x+\sin(p+d)x & \sin px & \sin(p+d)x \end{vmatrix}$$

$$\Rightarrow \qquad \Delta = \begin{vmatrix} 1+a^2 & a & a^2 \\ 2\cos px \cos dx & \cos px & \cos(p+d)x \\ 2\sin px \cos dx & \sin px & \sin(p+d)x \end{vmatrix}$$

Applying $C_1 \to C_1 - 2\cos dx\ C_2$

$$\Rightarrow \qquad \Delta = \begin{vmatrix} 1+a^2-2a\cos dx & a & a^2 \\ 0 & \cos px & \cos(p+d)x \\ 0 & \sin px & \sin(p+d)x \end{vmatrix}$$

$\Rightarrow \qquad \Delta = (1 + a^2 - 2a \cos dx)\ [\sin (p + d) x \cos px - \sin px\ \cos (p + d) x]$

$\Rightarrow \qquad \Delta = (1 + a^2 - 2a \cos dx) \sin dx$

Which is independent of *p*.

53.(A) Given,

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

Applying $C_3 \to C_3 - (C_1 + C_2)$

$$= \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1)(x-2) & 0 \end{vmatrix} = 0$$

$\therefore \qquad f(x) = 0 \Rightarrow f(100) = 0.$

54.(A) Since, a matrix is said to be singular, if $|A| = 0$

$$\Rightarrow \quad \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix} = 0$$

$$\Rightarrow \quad 8(0 - 12) - x(0 - 24) + 0(24 - 0) = 0$$

$$\Rightarrow \quad x = 4$$

55.(C) $\because A^2 = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence, A^2 is involutory.

56.(A) We have,

$$A^2 = A$$

$$\Rightarrow \quad A^2 - A = 0$$

$$\Rightarrow \quad m_A(x) = x^2 - x = x\,(x - 1)$$

57.(B)

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$[A - \lambda I] = 0$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix} = 0$$

$\Rightarrow \quad (2-\lambda)^2 = 0$

$\Rightarrow \quad \lambda = 2$

Now, consider the eigen value problem

$$[A - \lambda I]\hat{X} = 0$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Put $\lambda = 2$, we get,

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_2 = 0 \quad$ ... (i)

$0 = 0 \quad$ ... (ii)

The solution is therefore $x_2 = 0$, x_1 = anything

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k \\ 0 \end{bmatrix}$$

Since there is only one parameter in the infinite solution, there is only linearly independent eigen vector for this problem which may be written as $\begin{bmatrix} k \\ 0 \end{bmatrix}$ or as $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

58.(C) The characteristic roots are $|A - \lambda I| = 0$

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (1-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ -1 & -\lambda \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ -1 & -\lambda \end{vmatrix} - 2\begin{vmatrix} 1 & 2-\lambda \\ -1 & -1 \end{vmatrix} = 0$$

$\Rightarrow \quad (1 - \lambda) \{(2 - \lambda) X - \lambda + 1\} -2\{-\lambda + 1\} -2\{-1 + 2 - \lambda\} = 0$

$\Rightarrow \quad (1 - \lambda) [-2\lambda + \lambda^2 + 1] -2(1-\lambda) -2(1-\lambda) = 0$

$\Rightarrow \quad (1 - \lambda) [\lambda^2 - 2\lambda + 1 -4] = 0$

$\Rightarrow \quad \lambda = 1, -1, 3$

Now, eigen vectors

At $\lambda = 1 \Rightarrow [A - I] X_1 = 0$

$$\begin{bmatrix} 0 & 2 & -2 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x_2 - x_3 = 0 \Rightarrow x_2 = x_3$

$\Rightarrow \quad x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -2x_3$

$$\Rightarrow \quad X_1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

At $\lambda = -1 \Rightarrow [A + I] X_2 = 0$

$$\Rightarrow \quad \begin{bmatrix} 2 & 2 & -2 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \left.\begin{aligned} x_1 + x_2 - x_3 &= 0 \\ x_1 + 3x_2 + x_3 &= 0 \end{aligned}\right\}$$

$\Rightarrow \quad 2x_1 + 4x_2 = 0$

$\Rightarrow \quad 2x_2 = -x_1 \Rightarrow x_3 = x_2$

$$\Rightarrow \quad X_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

At $\lambda = 3 \quad \Rightarrow [A - 3]\, X_3 = 0$

$$\Rightarrow \quad \begin{bmatrix} -2 & 2 & -2 \\ 1 & -1 & 1 \\ -1 & -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \left.\begin{aligned} x_1 - x_2 + x_3 &= 0 \\ -x_1 - x_2 - 3x_3 &= 0 \end{aligned}\right\}$$

$\Rightarrow \quad -2x_2 - 2x_3 = 0$

$\Rightarrow \quad x_3 = -x_2$

$\Rightarrow \quad x_1 = 2x_2$

$$\Rightarrow \quad X_3 = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

So, model matrix

$$P = [X_1 \;\; X_2 \;\; X_3] = \begin{bmatrix} -2 & -2 & -2 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

So, diagonal matrix of

$$A = D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

59.(A) Given matrix is

$$A = \begin{bmatrix} 1 & a & a^2 & \dots & a^n \\ 1 & a & a^2 & \dots & a^n \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a & a^2 & \dots & a^n \end{bmatrix}_{(n+1)\times(n+1)}$$

Applying the following row operation, we get

$$R_2 \to R_2 - R_1$$

$$R_3 \to R_3 - R_1$$

$$R_4 \to R_4 - R_1$$

....

....

$R_n \rightarrow R_n - R_1$

$R_{n+1}\ R_{n+1} - R_1$

$$\sim \begin{bmatrix} 1 & a & a^2 & \ldots & a^n \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix}_{(n+1)\times(n+1)}$$

which is in echelon form and the number of non-zero rows is 1. Hence, $\rho(A) = 1$.

60.(C) $\because$ Given that $E \times F = G$

$$\Rightarrow \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow F = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

[Because the product of any matrix and unit matrix is the matrix itself.]

So, $$|E| = \cos\theta \begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} - (-\sin\theta) \begin{vmatrix} \sin\theta & 0 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} \sin\theta & \cos\theta \\ 0 & 0 \end{vmatrix}$$

$$= \cos^2\theta + \sin^2\theta = 1$$

Matrix of chfactors of E = C = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So, adjoint of E = C' = $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence, $$F = \frac{\text{adj}(E)}{|E|} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

61.(D) $\because$ The cross product of b = $[0\ \ 1\ \ 0]^T$ and x = $[x_1\ \ \ x_2\ \ \ x_3]^T$ is

$$b \times x = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ x_1 & x_2 & x_3 \end{vmatrix} = x_3 + 0j - x_1k$$

$$= [x_3\ \ \ 0\ \ \ -x_1]$$

Now, we have L(x) = b × X = M $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

where M is a 3 × 3 matrix

Let $M = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$

Now, $M\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = b = x$

$$\Rightarrow \quad \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ 0 \\ -x_1 \end{bmatrix}$$

By matching LHS and RHS, we get

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ 0 \\ -x_1 \end{bmatrix}$$

So, $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

Now, we have to find the eigen values of M as

$$|M - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 0 & 1 \\ 0 & -\lambda & 0 \\ -1 & 0 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad -\lambda(\lambda^2 - 0) + 1(0 - \lambda) = 0$

$\Rightarrow \quad \lambda^3 + \lambda = 0$

$\Rightarrow \quad \lambda = 0$ and $\lambda^2 = -1$

$\Rightarrow \quad \lambda^2 = \pm i$

Hence, $\quad \lambda = 0, \pm i$

So, the eigen values are 0, i, – i.

62.(B) Since, $\quad A \cap B = \{0\}$

$\Rightarrow \quad \dim(A + B) = \dim A + \dim B - \dim(A \cap B)$

$= 2 + 1 - 0 = 3$

$\therefore \quad \dim(A + B) = \dim V$

$\Rightarrow \quad V = A + B$

63.(B) Since, total number of variable -dependent variables = 4 –2 = 2

Hence, $\quad \dim V = 100 - 2 = 98$

64.(A) $\because f(A) = A^2 - 5A + 6I$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

65.(B) $\because X = [X1\ X_2 \ldots X_n]'$

$$V = XX' = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}[X_1X_2.....X_n]$$

$$= \begin{bmatrix} X_1^2 & X_1X_2 & X_1X_3 & \cdots & X_1X_n \\ X_2X_1 & X_2^2 & X_2X_3 & \cdots & X_2X_n \\ X_3X_1 & X_3X_2 & X_3^2 & \cdots & X_3X_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X_nx_1 & X_nX_2 & X_nX_3 & \cdots & X_n^2 \end{bmatrix}$$

Now, If we perform following elementary operations on above

$(\because X_1, X_2, ..., X_n \neq 0)$

$$R_2 \to \frac{R_2}{x_2} - \frac{R_1}{x_1}$$

$$R_3 \to \frac{R_3}{x_3} - \frac{R_1}{x_1}$$

....

$$R_n \to \frac{R_n}{x_n} - \frac{R_1}{x_1}$$

We get $\begin{bmatrix} x_1^2 & x_1x_2 & x_1x_3 & \cdots & x_1x_n \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$

Hence rank of matric = 1

66.(D) In an over determined system having more equations than variables all three possibilities can exist

(a) Consistent and unique, if r = n

(b) Consistent and infinite, if Rank of $\bar{A}$ = Rank of A $\neq$ number of unknowns

(c) incosistent and no solution, if Rank of |A : B| $\neq$ Rank of [A].

67.(B) For matrix A^m, m being a positive integer $\left(\lambda_i^m, X_i^m\right)$ will be the eigen pair for all *i*.

Although λ_i^m will be the corresponding eigen values of A^m, X_i^m will not be corresponding eigen vectors.

68.(A) $\because$ If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of *A*, then the eigen values of A^m are $\lambda_1^m, \lambda_2^m, \ldots, \lambda_n^m$.

$\because$ S has eigen values 1 and 5, so S^2 has eigen values 1^2 and 5^2.

$\Rightarrow$ 1 and 25

69.(C) The Augmented matrix

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right]$$

Performing gauss elimination on [A|B] we get

$$= \left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_3 - \frac{1}{2}R_1} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right] \xrightarrow{R_3 - \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Rank (A) = Rank (A|B) = 2 < 3

So infinite number of solutions are obtained.

70.(B) The augmented matrix for the system of equations is

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array}\right]$$

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 0 & 0 & \lambda - 6 & \mu - 20 \end{array}\right] \qquad [R_3 \to R_3 - R_2]$$

If λ = 6 and $\mu \neq 20$ then

Rank (A|B) = 3 and Rank (A) = 2

$\because$ Rank (A|B) $\neq$ Rank (A)

$\therefore$ Given system of equations has no solution for λ = 6 and $\mu \neq 20$.

71.(C) $M = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}, [M - \lambda I] = \begin{bmatrix} 4-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}$

Given eigen vector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$

$$[M - \lambda I]\hat{X} = 0$$

$\Rightarrow \quad \begin{bmatrix} 4-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}\begin{bmatrix} 101 \\ 101 \end{bmatrix} = 0$

$\Rightarrow \quad (4 - \lambda)(101) + 2 \times 101 = 0$

$\Rightarrow \quad \lambda = 6$

72.(A) If $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_4$ are the eigen values of A. Then the eigen values of

Am are $\lambda_1^m, \lambda_2^m, \lambda_3^m ....$

Here, S matrix has eigen values 1 and 5.

So, S^2 matrix has eigen values 1^2 & 5^2 i.e. 1 and 25.

73.(C) (i) The Eigen values of symmetric matrix [$A^T = A$] are purely real.

(ii) The Eigen value of skew-symmetric matrix [$A^T = -A$] are either purely imaginary or zeros.

74.(C) $\Sigma\lambda_i$ = Trace (A) = –2 $\Rightarrow \lambda_1 + \lambda_2 = -2$ ... (i)

$\prod \lambda_i = |A| = -35 \Rightarrow \lambda_1\lambda_2 = -35$ ... (ii)

Solving (i) and (ii) we get λ_1 and λ_2 = 5, –7

75.(A) The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 3 \\ 4 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (1 - \lambda)(2 - \lambda) - 12 = 0$$

$$\Rightarrow \quad \lambda^2 - 3\lambda - 10 = 0$$

By Cayley-Hamilton theorem

$$A^2 - 3A - 10 \, | = 0$$

$$\Rightarrow \quad I = \frac{1}{10}\left[A^2 - 3A\right]$$

Pre-multiplying by A–1 we get

$$A^{-1} = \frac{1}{10}[A - 3I] = \frac{1}{10}\left(\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)$$

$$= \frac{1}{10}\begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & \frac{3}{10} \\ \frac{2}{5} & -\frac{1}{10} \end{bmatrix}$$

76.(C) Given differential equation $(D^2 + bD + C)y = 0$

if $y = xe^{-5x}$ is one of its solutions

then $Dy = e^{-5x} - 5xe^{-5x}$

$D^2y = -10e^{-5x} + 25xe^{-5x}$

$(D^2 + bD + C)\,y \quad = (10 + 25x)e^{-5x} + b(1 - 5x)e^{-5x} + cxe^{-5x}$

$\qquad = e^{-5x}\,[(10 - b) + (25 - 5b + c)x]$

$\Rightarrow \quad 10 + b = 0$

$\Rightarrow \quad 25 - 5b + c = 0$

$\Rightarrow \quad \begin{cases} b = -10 \\ c = 25 \end{cases} \Rightarrow$ b is negative c is positive

77.(D) Given set V = $\{(x_1 - x_2 + x_3, x_1 + x_2 - x_3) : (x_1, x_2, x_3) \in \mathbb{R}^3\}$

Let for (x_1, x_2, x_3), (x_1', x_2', x_3')

Let $(x_1 - x_2 + x_3, x_1 + x_2 - x_3)$, $(x_1' - x_2' + x_3', x_1' + x_2' - x_3') \in V$

for α, β

$\alpha(x_1 - x_2 + x_3, x_1 + x_2 - x_3) + \beta(x_1' - x_2' + x_3' - x_1' + x_2' - x_3')$

$= (\alpha(x_1 - x_2 + x_3) + \beta(x_1' - x_2' + x_3'), \alpha(x_1 + x_2 - x_3)) + \beta(x_1' + x_2' - x_3') \in V$

$\Rightarrow$ V is subspace

and for (1, 1, 1) $\in \mathbb{R}^3$ we get (1, 1) $\in$ V

and for (1, 1, 0) $\in \mathbb{R}^3$ we get (0, 2) $\in$ V

and {(1, 1), (0, 2)} is LI set

$\Rightarrow$ dim V = 2 $\Rightarrow$ dim V = dim $\mathbb{R}^2$

and $V \subseteq \mathbb{R}^2$

$\Rightarrow \quad V = \mathbb{R}^2$

78.(D) Since A be a 5 × 5 matrix all of whose eigenvalues are zero

$\Rightarrow$ by calay hamilton's theorem

The characteristic equation for matrix A is given by (A – 0) (A – 0) (A – 0) (A – 0) = 0

$\Rightarrow$ $(A - 0)^5 = 0$

$\Rightarrow$ $A^5 = 0$

79.(B) Given series $\sum_{n=0}^{\infty} z^{n!}$

$$= \sum_{n=0}^{\infty} z^{1.2.3......n}$$

here $a_{n!} = 1$

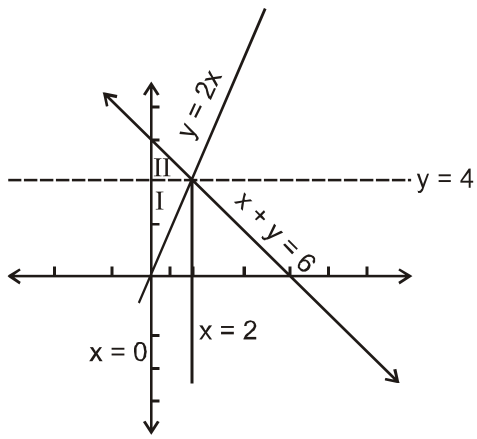
$a_{n!+1} = 1$

so radius of convergence $= \frac{1}{R} = \lim_{n\to\infty} \frac{a_{n!}}{a_{n!+1}}$

$$= \frac{1}{R} = 1$$

$$\Rightarrow \quad R = 1$$

80.(B)



Since Let $I = \int_0^2 \int_{2x}^{6-x} f(x,y)\,dy\,dx$

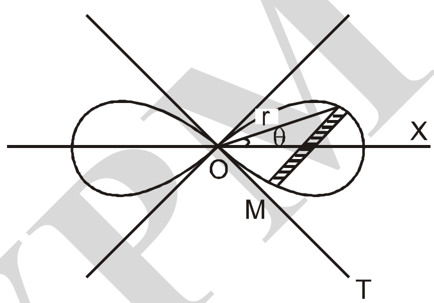
on changing order of integration

$$I = \iint_I f(x,y)\,dx\,dy + \iint_{II} f(x,y)\,dx\,dy$$

so $\quad I = \int_0^4 \int_0^{y/2} f(x,y)\,dx\,dy + \int_4^6 \int_0^{6-y} f(x,y)\,dx\,dy$

81.(C) Let P(r, θ) be a point on the lemniscate

$$r^2 = a^2 \cos 2\theta$$



Let OT be a tangent at the pole

$\therefore \quad \angle POM = \theta + \frac{\pi}{4}$

PM = OP isn POM

$$= r \sin\left(\theta + \frac{\pi}{4}\right)$$

$$= a\sqrt{\cos 2\theta}\sin\left(\theta + \frac{\pi}{4}\right)$$

$\therefore$ The required surface area

$$S = 2\int_{-\pi/4}^{\pi/4} 2\pi . PM\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= 4\pi\int_{-\pi/4}^{\pi/4} a\sqrt{\cos 2\theta}\sin\left(\theta + \frac{\pi}{4}\right) \times \sqrt{a^2\cos 2\theta + \frac{a^4\sin^2 2\theta}{r^2}}$$

$$= 4\pi\int_{-\pi/4}^{\pi/4} a\sqrt{\cos 2\theta \sin}\left(\theta + \frac{\pi}{4}\right) \times \frac{a}{\sqrt{\cos 2\theta}}\, d\theta$$

$$= 4\pi a^2\int_{-\pi/4}^{\pi/4} \sin\left(\theta + \frac{\pi}{4}\right) d\theta$$

$$= 4\pi a^2\left[-\text{cis}\left(\theta + \frac{\pi}{4}\right)\right]_{-\pi/4}^{\pi/4}$$

$= 4\pi a^2$

82.(C) Since $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

if it's characteristic roots are I, I so by a well known result det A = multiplication of eigen values

det A = λ^2

if λ be any real no.

then det (A) $\geq 0 \quad \forall \lambda \in \mathbb{R}$

83.(B) The given differential equation can be written as

$$\frac{dy}{dx} + iy = x$$

Integrating factor $e^{\int i dx} = e^{ix}$

and solution $ye^{ix} = \int xe^{ix}\, dx + c$

$$ye^{ix} = x\frac{e^{ix}}{i} - \frac{e^{ix}}{(-1)} + c$$

$$y = -ix + 1 + ce^{-ix}$$

$\phi(x) = -ix + 1 + ce^{-ix}$

$\phi(0) = 1 + c = 2$

$\Rightarrow \quad c = 1$

so $\phi(\pi) = -i\pi + 1 - 1$

$= -i\pi$

84.(A) Since given ODE $\quad x^2y' + 2xy = 1$

$$y' + \frac{2x}{x^2}y = \frac{1}{x^2}$$

$$y' + \frac{2}{x}y = \frac{1}{x^2}$$

which is a linear equation then

$$IF = e^{\int 2/x\,dx} = -e^{\log x^2}$$

$$= x^2$$

solution is given by $\quad yx^2 = \int x^2 \frac{1}{x^2}dx + c$

$yx^2 = +x + c$ **Ans.**

$\phi(x) = y = \frac{1}{x} + \frac{c}{x^2}$

as $x \to \infty \qquad y \to 0$

85.(B) Since given $\quad (f(x))^2 = 1 + 2\int_0^x f(t)dt$

on differentiating w.r.to. x on both sides

we get $2f(x)\frac{df}{dx} = 2(x)$

$\Rightarrow \qquad 2f(x)\left[\frac{df}{dx} - 1\right] = 0$

$\Rightarrow \qquad \frac{df}{dx} - 1 = 0 \qquad$ (as $f(x) \neq 0 \ \forall x$)

$\Rightarrow \qquad f = x + c$

for our convenience we can take c = 0

$\Rightarrow \qquad f(x) = x$

$f(1) = 1$

86.(B) Here V = $\{(x_1, x_2, x_3) : x_1^2 + x_2^2 + x_3^2 = k$, $x_1, x_2, x_3 \in R\}$

for being subspace of $\mathbb{R}^3$ k should not be –1 otherwise one of x_1 x_2 x_3 becomes complex and k should not be zero otherwise

$x_1^2 + x_2^2 = -x_3^2$ again we get any of x_1, x_2, x_3 complex values k can be 1

so that $x_1^2 + x_2^2 + x_3^2 = 1$

and v be the sphere of radius 1 in $\mathbb{R}^3$

87.(B) Since a linear transformation $T : R^3 \to R^2$

defined as $Tv_1 = v_2$ $Tv_2 = v_3$ $Tv_3 = v_1$

$\Rightarrow$ T is cyclic linear transformation

where $v_1 = (1, 0)$ $v_2 = (1,-1)$ $v_3 = (0, 1)$

here by $\{v_1\ v_2\ v_3\}$ we can get three sets of basis for R^2

$\{v_1\ v_2\}$ $\{v_2\ v_3\}$ $\{v_3\ v_1\}$

if we take $\{v_1, v_2\}$ as the basis ad to determine linear transformation.

$(x, y) = \alpha(1, 0) + \beta(1, -1)$

$x = a + b$

$y = -b$ $\quad T(x, y) = \alpha T(1, 0) + \beta T(1, -1)$

$x + y = a$ $\quad = \alpha(1, -1) + \beta(0, 1)$

$T(x, y) = (x + y, x - 2y)$

if we take $\{v_2\ v_3\}$ as the basis

on determining linear transformation

$(x, y) = \alpha(1, -1) + \beta(0, 1)$

$\Rightarrow$ $x = a$ $\quad y = -a + b$

$x = a$ $\quad y + x = b$

$\Rightarrow$ $T(x, y) = \alpha T(1, -1) + \beta T(0, 1),$

$= \alpha(0, 1) + \beta(1, 0)$

$T(x, y) = (b, \alpha)$

$T(x, y) = (x + y, x)$

Similarly we can get a linear transformation for $\{v_3, v_1\}$

$\Rightarrow$ 3 possible such linear transformation can be there

88.(B) here let n be the order of a group G and let $a \in G$ be any element of G of order m

so $\quad a^{10(G)} = e$ (identity)

$\Rightarrow$ $\quad a^n = e$

or $a^m = e$ (identity)

obviously $m \le n$

but so m can be a divisor of n .... (1)

But $a^{nk} = e$ k be any integer

$\Rightarrow$ any multiple of order of group also satisfies the condition ... (2)

By statement (1) and (2)

Hence the number involving any of factor (2 or 5) could be the order of group.

so 9 = 3 × 3 does not contain any factor

$\Rightarrow$ it can not be the order of group Gs.t. $a \in G$ $a^{20} = e$

89.(C) If the required matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \ldots\ldots & 0 \\ 0 & 1 & 0 \ldots\ldots & \Theta \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ 0 & 0 & 0 \ldots\ldots & 1 \end{bmatrix}_{10\times 10} \quad \text{s.t. } |A| = 1$$

The required type of matrices can be formed by interchanging row and columns under restrictions

so determinant of such type of matrices could be 1 or –1

some times it could be zero

but values of determinant could not possible is 10.

90.(A) Since it is a well known fact that every principal ideal domain is a euclidean domain

$\Rightarrow$ statement (I) is true

But all the units of ring Z/372 cannot form cyclic group as we can not find a generator.

$\Rightarrow$ statement (II) is false

now let p be a prime and $n \ge 1$ be an integer then there exists a field with p^n element

Since 6 is not a prime number

$\Rightarrow$ there does not exists a field with 6^5 elements

91.(B) permutation group on 1, 2, 3........n is S_n every element of S_n can be represented as the product of disjoint of cycles as well as transpositions

S_n is a group generated by (1, 2) and (1, 2, ...,n) there may be more generators But (1, 2) (3,4) is not conjugate to (1, 2) (1, 3) because they are not commutative to each other.

92. (B) Since $T_1, T_2 : V_3(R) \to V_3(R)$

Since $T_1(x_1, x_2, x_3) = (0, 0, x_2)$

$T_1(T_1(x_1,x_2,x_3) = T(0, 0, x)$

$T_1^2(x_1, x_2, x_3) = (0,0,0)$

$\Rightarrow$ T_1 is nilpotent of order 2

$T_2(x_1,x_2, x_3) = (x_1, 0, 0)$

$T_2(T_2(x_1, x_2, x_3) = T(x_1, 0, 0) = (x_1, 0, 0)$

$\Rightarrow$ $T_2^2(x_1, x_2, x_3) = (x_1, 0,0)$

$\Rightarrow$ T_2 is idempotent linear transformation.

93. (A) Since linear transformation is

defined as $T : R^4 \to R^3$

s.t. T(x, y, z, w) = (x – y + z + w, x + 2z – w, x + 3z – 3w)

first we find out the matrix corresponding to standard basis.

T(1, 0, 0, 0) = (1, 1, 1) = 1(1, 0, 0) + 1(0, 1, 0) + 1(0, 0, 1)

T(0, 1, 0, 0) = (–1, 0, 1) = –1(1, 0, 0) + 0(0, 1, 0) + 1(0, 0, 1)

T(0, 0, 1, 0) = (1, 2, 3) = 1(1, 0, 0) + 2(0, 1, 0) + 3(0, 0, 1)

T(0, 0, 0, 1) = (1, –1, –3) = 1(1, 0, 0) + (–1) (0, 1, 0) + (–3) (0, 0, 1)

so coefficient are $[1\ \ 1\ \ 1]^T$ $[-1\ ,\ 0\ \ 1]^T$ $[1\ \ 2\ \ 3]^T$ $[1\ \ -1\ \ -3]^T$

so required matrix $A = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 0 & 2 & -1 \\ 1 & 1 & 3 & 3 \end{bmatrix}$

we know that rang T = rank of matrix correspond to T

By using elementary row and column transformations on changing A into echelon form.

$$A \sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & 4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$A \sim \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix} \qquad R_3 \to \frac{R_3}{2}$$

$$A \sim \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & -2 \\ 0 & 1 & 1 & 2 \end{bmatrix} \qquad R_2 \to R_2 - R_3$$

$\Rightarrow$ Rang(A) = 3

$\Rightarrow$ Rang T = 3

94.(C) Since sequence $\left\langle \frac{(-1)^n}{n^2} \right\rangle$ is alternating so its alternate terms are positive it can be splitted into two sequences $\left\{-1, \frac{-1}{9}, \frac{-1}{25} \ldots\ldots \right\}$ and $\left\{\frac{1}{4}, \frac{1}{16}, \frac{1}{36} \ldots\ldots \right\}$

The limit point of first sequence gives lower bound and limit of second sequence gives upper bound

here limit of both sequences are zero $\Rightarrow$ limit superior and limit inferior are both zero

95.(C) Suppose that $<s_n>$ converges take $\varepsilon = \frac{1}{2}$ by cauchy convergence criterion there must exist $m \in N$ such that

$$|S_n - S_m| < \frac{1}{2} \text{ for all } n \geq m$$

i.e. $$\left| \frac{1}{2m+1} + \frac{1}{2m+3} + \ldots\ldots + \frac{1}{2n-1} \right| < \frac{1}{2} \qquad \forall n \geq m$$

take n = 2m

$$\left| \frac{1}{n+1} + \frac{1}{n+3} + \ldots\ldots + \frac{1}{2n-1} \right| < \frac{1}{2}$$

$$\left(\frac{n/2}{n+1} \right) = \frac{1}{2} \text{ as } n \to \infty \ \forall n$$

Hence we get a contradiction therefore the sequence cannot converge i.e. not cauchy

96.(C) Since $f(x,y) = \begin{cases} x^2 + y^2, & \text{if } x \text{ and } y \text{ rational} \\ 0, & \text{otherwise} \end{cases}$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)(0,0)} \left(x^2 + y^2\right) = f(0,0)$$

$\Rightarrow$ f is continuous at (0, 0)

now $\quad f_x(0,0) = \lim_{h\to 0} \dfrac{f(0+h,0) - f(0,0)}{h} = \lim_{h\to 0} h = 0$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,0+k) - f(0,0)}{k} = \lim_{k\to 0} k = 0$$

$$f_x(0, 0) = f(0, 0) = f_y(0, 0)$$

$\Rightarrow$ f is differentiable at (0, 0)

97.(C) Let $y = e^x$ be one of the solution of

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} + (1-x)y = 0 \qquad .... (1)$$

then let its solution is given by $y = ve^x$

on satisfying (1) by $y = ve^x$

$$e^x x\left[v + 2\frac{dv}{dx} + \frac{d^2v}{dx^2}\right] - \left[v + \frac{dv}{dx}\right]e^x + (1-x)ve^x = 0$$

$$xv + 2x\frac{dv}{dx} + x\frac{d^2v}{dx^2} - v + \frac{dv}{dx} + v - xv = 0$$

$$x\frac{d^2v}{dx^2} + (2x+1)\frac{dv}{dx} = 0$$

Let $\dfrac{dv}{dx} = p$

$$\Rightarrow \qquad x\frac{dp}{dx} + (2x+1)p = 0$$

$$\Rightarrow \qquad \frac{dp}{p} = \frac{-(2x+1)}{x}dx$$

on integrating both sides

$\log p = -2x + \log x + \log c$

$p = c_1 x e^{-2x}$

$$\frac{dv}{dx} = c_1 x e^{-2x}$$

$dv = c_1 x e^{-2x}\, dx$

again integrating

$$v = c_2 + c_1\left[x\frac{e^{-2x}}{-2} - \frac{e^{-2x}}{4}\right]$$

$$v = c_2 + \frac{c_1}{(-2)}\left[xe^{-2x} + \frac{e^{-2x}}{2}\right]$$

$$v = c_2 \frac{-c_2}{2} e^{-2x}\left(x + \frac{1}{2}\right)$$

$$e^x v = c_2 e^x - \frac{c_1}{2}\left(x + \frac{1}{2}\right)$$

$$t = c_2 e^x - \frac{c_1}{2} e^{-x}\left(x + \frac{1}{2}\right)$$

second linearly independent solution of this ODE is $-\left(x + \frac{1}{2}\right)\frac{e^{-x}}{2}$.

98.(A) Given differential equation.

$(xD^2 + D + x)y = 0,\quad y(0) = 1$

$y'(0) = 0$

on multiplying by x on both sides

we get a homogeneous differential equation

$(x^2D + xD + x^2)y = 0$

if Let $z = \log x$

$D = \frac{d}{dx}$ $\qquad$ $D' = \frac{d}{dz}$

$\Rightarrow \quad x^2D^2 = D'(D' - 1) \qquad D' = xD$

so $\quad [D'(D' - 1)] + D + e^{2z})y = 0$

$[D'^2 + e^{2z}]y = 0$

the roots of auxiliary equation is $D' = \pm ie^z$

Sol. is given by

$y = c_1 \sin e^z + c_2 \cos ez$

$y = c_1 \sin x + c_2 \cos x$

$y' = c_1 \cos x - c_2 \sin x$

$y(0) = 1 = c_2$

$y'(0) = 0 = c_1$

so y = cos x a unique solution.

99.(D) $x^2 + y^2 = 3a^2$ ... (i)

$x^2 = 2ay$ ... (ii)

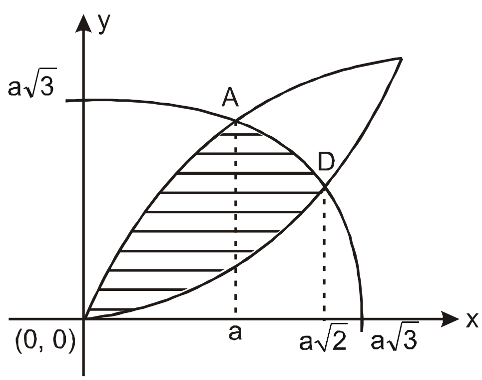
$y^2 = 2ax$ ... (iii)

Solving (i) and (ii)

$$2ay + y^2 = 3a^2$$

$$x^2 + 2ax - 3a^2 = 0$$

Hence, we get the only positive root $x_A = a$.



Analogously we find the abscissa of the point D of intersection of the circle $x^2 + y^2 = 3a^2$ and the parabola

$x^2 = 2ay$ $\quad x_0 = a\sqrt{2}$

$$S = \int_0^{a\sqrt{2}} \left[y_2(x) - y_1(x)\right] dx$$

Hence $y_1(x) = \dfrac{x}{2a}, \quad y_2(x) = \begin{cases} \sqrt{2ax} & \text{for } 0 \le x \le a \\ 3a - x & \text{for } a < x \le a\sqrt{2} \end{cases}$

By Additive property of integral

$$S = \int_0^a \left(\sqrt{2ax} - \frac{x^2}{2a}\right) dx + \int_0^{a\sqrt{2}} \left(\sqrt{3a^2 - x^2} - \frac{x^3}{2a}\right) dx$$

$$= \left|\sqrt{2a} \cdot \frac{2}{3} \cdot x^{3/2} - \frac{x^3}{6a}\right|_0^a + \left[\frac{x}{2} \cdot \sqrt{3a^2 - x^2} + \frac{3a^3}{3} \sin^{-1} \frac{x}{a\sqrt{3}} - \frac{x}{6a}\right]_a^{a\sqrt{2}}$$

$$= \left(\frac{\sqrt{2}}{3} + \frac{3}{2} \sin^{-1} \frac{1}{3}\right) a^2$$

100.(B) Given differential equation is

$$\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$$

$$\Rightarrow \qquad \frac{1}{z(\log z)^2}\frac{dz}{dx}+\frac{1}{x\log z}=\frac{1}{x^2}$$

$$\text{Let } \frac{1}{\log z}=t$$

$$\Rightarrow \qquad -(\log z)^{-2}\times\frac{1}{z}\frac{dz}{dx}=\frac{dt}{dx}$$

$$\Rightarrow \qquad \frac{1}{z(\log z)^2}\frac{dz}{dx}=-\frac{dt}{dx}$$

$$\Rightarrow \qquad -\frac{dt}{dx}+\frac{t}{x}=\frac{1}{x^2}$$

$$\Rightarrow \qquad \frac{dt}{dx}+P(x)t=Q(x)$$

Here $P(x)=\frac{-1}{x}$ and $Q(x)=\frac{-1}{x^2}$ which shows that the transformation $t=\frac{1}{\log z}$ reduce the given differential equation into the form $\frac{dt}{dx}+P(x)t=Q(x)$.

101.(B) Since β_1 is parallel to α,

Let $\beta_1=\lambda\alpha$, where λ is a scalar.

$\beta=\beta_1+\beta_2$ (Given)

$$=2\hat{i}+\hat{j}-3\hat{k}-\lambda\alpha=2\hat{i}+\hat{j}-3\hat{k}-\lambda\left(3\hat{i}-\hat{j}\right)$$

$$=\left(2-3\lambda\right)\hat{i}+\left(1+\lambda\right)\hat{i}-3\hat{k}$$

Since β_2 is perpendicular to α.

$\therefore \quad \beta_2 \cdot \vec{a} = 0$

$\Rightarrow \quad \left[(2-3\lambda)\hat{i} + (1+\lambda)\hat{i} - 3\hat{i}\right] \cdot \left(3\hat{i} - \hat{i}\right) = 0$

$\Rightarrow \quad 3(2 – 3\lambda) – (1 + \lambda) = 0$

$\Rightarrow \quad 6 – 9\lambda – 1 – \lambda = 0$

$\Rightarrow \quad 5 – 10\lambda = 0, \therefore \lambda = \dfrac{1}{2}.$

$\therefore \quad \beta_1 = \lambda\alpha = \dfrac{1}{2}\left(3\hat{i} - \hat{i}\right) = \dfrac{3}{2}i - \dfrac{1}{2}\hat{j}$

$\beta_2 = \left(2 - \dfrac{3}{2}\right)\hat{i} + \left(1 + \dfrac{1}{2}\right)\hat{k} - 3\hat{k} = \dfrac{1}{2}\hat{i} + \dfrac{3}{2}\hat{j} - 3\hat{k}$

$\therefore \quad 2\hat{i} + \hat{j} - 3\hat{k} = \lambda\alpha + \beta_2$

Hence $\beta = \beta_1 + \beta_2$

102.(C) Given $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}; \hat{b} = -i - 2\hat{j} + 4\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ -1 & -2 & 4 \end{vmatrix}$$

$= \hat{i}(-4+4) - \hat{j}(12+2) + \hat{k}(-6-1) = -14\hat{j} - 7\hat{k}$

A unit vector along $\vec{a} \times \vec{b} = \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$

$$= \frac{-14\hat{j} - 7\hat{k}}{\sqrt{(14)^2 + (7)^2}} = \frac{-14\hat{j} - 7\hat{k}}{\sqrt{245}} - \frac{-14j - 7\hat{k}}{\sqrt{49 \times 5}}$$

$$= \frac{-14\hat{j} - 7\hat{k}}{7\sqrt{5}} = \frac{-2\hat{j} - \hat{k}}{\sqrt{5}}.$$

103.(C) We have,

$$\overrightarrow{OA} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right); \overrightarrow{OB} = \hat{i} - \hat{j} - 3\hat{k}.$$

$$\overrightarrow{OC} = 4\hat{i} - 3\hat{j} + \hat{k}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -2\hat{i} - 5\hat{k}$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \hat{i} - 2j - \hat{k}.$$

The vector $\overrightarrow{AB} \times \overrightarrow{AC}$ is $\perp$ to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and consequently to the plane ABC.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= i(0 - 10) - j(2 + 5) + \hat{k}(4 - 0)$$

$$= -10\hat{i} - 7\hat{j} + 4\hat{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \left|-10\hat{i} - 7\hat{j} + 4\hat{k}\right|$$

$$= \sqrt{(-10)^2 + (-7)^2 + 4^2}$$

$$= \sqrt{100 + 49 + 16} = \sqrt{165}$$

$\therefore$ A unit vector $\perp$ to the plane ABC

$$= \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|} = \frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{165}}$$

$$= -\frac{10}{\sqrt{165}}\hat{i} - \frac{7}{\sqrt{165}}\hat{j} + \frac{4}{\sqrt{165}}\hat{k}.$$

104.(C) A vector which is $\perp$ to $\vec{a}$ as well as $\vec{b}$ is given by $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$$\vec{a}\times\vec{b} = \left(3\hat{i}+\hat{j}-4\hat{k}\right)\times\left(6\hat{i}+5\hat{j}-2\hat{k}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix}$$

$$= (-2+20)\hat{i} - (-6+24)\hat{j} + (15-6)\hat{k}$$

$$= 18\hat{i} - 18\hat{j} + 9\hat{k} = 9\left(2\hat{i}-2\hat{j}+\hat{k}\right).$$

$\therefore$ A unit vector $\perp$ to both $\vec{a}$ and $\vec{b}$ is

$$\eta = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{9\left(2\hat{i}-2\hat{j}+\hat{k}\right)}{9\sqrt{4+4+1}} = \frac{2i-2\hat{j}+\hat{k}}{3}$$

Hence the required vector

$$= 3(\eta) = 3\left(\frac{2\hat{i}-2\hat{j}+\hat{k}}{3}\right) = 2\hat{i}-2\hat{j}+\hat{k}.$$

105.(A) Let $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\therefore \qquad \vec{R}\cdot\vec{A} = 0 \Rightarrow 2x + z = 0 \qquad \text{... (1)}$$

$$\vec{R}\times\vec{B}=\vec{C}\times\vec{B}\Rightarrow\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ 1 & 1 & 1\end{vmatrix}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -3 & 7\\ 1 & 1 & 1\end{vmatrix}$$

$\Rightarrow \quad (y-z)\hat{i}+(z-x)\hat{j}+(x-y)\hat{k}=-10\hat{i}+3\hat{j}+7\hat{k}$

$\Rightarrow \quad y - z = -10$ ... (2)

$z - x = 3$ ... (3)

and $x - y = 7$.

Solving (1) and (2), we get $x = -1$, $z = 2$.

$\therefore$ From (2), $y = -8$.

Hence $\vec{R}=-\hat{i}-8\hat{j}+2\hat{k}$.

106.(A) The vector area of the triangle ABC

$$=\frac{1}{2}\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)$$

Its modulus $=\frac{1}{2}$ base AB × Perp. distance of C from AB

i.e., $\frac{1}{2}\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|=\frac{1}{2}\left|\vec{b}-\vec{a}\right|\times p$

$\left[\because \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=b-a, \therefore \left|\overrightarrow{AB}\right|=\left|\vec{b}-\vec{a}\right|\right]$

Hence, $p=\dfrac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|}{\left|\vec{b}-\vec{a}\right|}$.

107.(C) Here $\vec{A},\vec{B},\vec{C}$ are the vectors which represent the sides of the triangle ABC where

$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{B} = d\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{C} = 3\hat{i} + \hat{j} - 2\hat{k}$

Given that, $\vec{A} = \vec{B} + \vec{C}$

$\therefore$ $a\hat{i} + b\hat{j} + c\hat{k} = (d+3)\hat{i} + 4\hat{j} + 2\hat{k}$

$\Rightarrow$ a = d + 3, b = 4, c = 2.

$$\vec{B} \times \vec{C} = \begin{vmatrix} i & j & k \\ d & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix}$$

$$= -10\hat{i} + (2d+12)\hat{j} + (d-9)\hat{k}$$

$\therefore$ Area of the $\Delta ABC = \frac{1}{2}\left|\vec{B} \times \vec{C}\right|$

$$= \frac{1}{2}\sqrt{\left[100 + (2d+12)^2 + (d-9)^2\right]}$$

$= 5\sqrt{6}$ (Given)

$\Rightarrow$ $\sqrt{(5d^2 + 30d + 325)} = 10\sqrt{6}$

$\Rightarrow$ $5d^2 + 30d + 325 = 600 \Rightarrow 5d^2 + 30d - 275 = 0$

$\Rightarrow$ $d^2 + 6d - 55 = 0$

$\Rightarrow$ (d + 11) (d – 5) = 0

$\Rightarrow$ d = 5 or –11

When d = 5, a = 8, b = 4, c = 2

and when d = –11, a = –8, b = 4, c = 2.

108.(A) We have, $\vec{F} = 4\hat{i} + 2\hat{j} + \hat{k}$

Let O be the point $3\hat{i} - \hat{j} + 3\hat{k}$ and P be point $5\hat{i} + 2\hat{j} + 4\hat{k}$.

$\therefore \quad \overrightarrow{OP} = \left(5\hat{i} + 2\hat{j} + 4\hat{k}\right) - \left(3\hat{i} - \hat{j} + 3\hat{k}\right)$

$= 2\hat{i} + 3\hat{j} + \hat{k} = \vec{r}\,(\text{say})$

Torque (vector moment) of $\vec{F}$ about O

$$= \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 4 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(3-2) - \hat{j}(2-4) + \hat{k}(4-12) = \hat{i} + 2\hat{j} - 8\hat{k}.$

109.(A) Vector perpendicular to face OAB

$= \overrightarrow{OA} \times \overrightarrow{OB}$

$= \left(\hat{i} + 2\hat{j} + \hat{k}\right) \times \left(2\hat{i} + \hat{j} + 3\hat{k}\right)$

$= 5\hat{i} - \hat{j} - 3\hat{k}$

Vector perpendicular to face ABC

$\overrightarrow{AB} \times \overrightarrow{AC}$

$= \left(\hat{i} - j + 2\hat{k}\right) \times \left(-2\hat{i} - \hat{j} + \hat{k}\right) = \hat{i} - 5\hat{j} - 3\hat{k}$

Since angle between faces equals angle between their normals. Therfore

$$\cos\theta = \frac{5 \times 1 + (-1) \times (-5) + (-3) \times (-3)}{\sqrt{5^2 + (-1)^2 + (-3)^2}\ \sqrt{1^2 + (-5)^2 + (-3)^2}}$$

$$= \frac{5+5+9}{\sqrt{35}\ \sqrt{35}} = \frac{19}{35}$$

$$\theta = \cos^{-1}\left(\frac{19}{35}\right).$$

110.(C) Let $\vec{a} = \vec{u} + \vec{w}, \vec{b} = \vec{u} \times \vec{v}$ and $\vec{c} = \vec{v} \times \vec{w}$

Then,

$$\vec{a} = \vec{u} + \vec{w} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right) + \left(\hat{j} - \hat{k}\right) = -\hat{i} - \hat{j},$$

$$\vec{b} = \vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & 1 \\ 3 & 0 & 1 \end{vmatrix} = -2\hat{i} + 4\hat{j} + 6\hat{k}$$

and $$\vec{c} = \vec{u}.\vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + 3\hat{j} + 3\hat{k}$$

$\therefore$ $$\left(\vec{u} + \vec{w}\right) \cdot \left[\left(\vec{u} \times \vec{v}\right) \times \left(\vec{v} \times \vec{w}\right)\right]$$

$$= \vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = \begin{vmatrix} -1 & -1 & 0 \\ -2 & 4 & 6 \\ -1 & 3 & 3 \end{vmatrix} = -1(-6) + 1 \cdot (0) + 0 = 6.$$

111.(C) We have, $\vec{i} \times \left(\vec{a} \times \vec{i}\right) + \vec{j} \times \left(\vec{a} \times \vec{j}\right) + \vec{k} \times \left(\vec{a} \times \vec{k}\right)$

$$= \left[\left(\vec{i} \cdot \vec{i}\right)\vec{a} - (i \cdot \vec{a})\hat{i}\right] + \left[\left(\hat{j} \cdot \hat{j}\right)\vec{a} - \left(\hat{j} \cdot \hat{a}\right)\hat{j}\right] + \left[\left(\hat{k} \cdot \hat{k}\right)\vec{a} - \left(\hat{k} \cdot \hat{a}\right)\hat{k}\right]$$

$= a - \left(\hat{i} - \vec{a}\right) + \vec{a} - \left(\hat{j} \cdot \vec{a}\right)\hat{j} + \vec{a} - \left(\hat{k} \cdot \vec{a}\right)\hat{k} \qquad \left[\because i^2 = j^2 = k^2 = 1\right]$

$= 3\vec{a} = \left[\left(\hat{i} \cdot \vec{a}\right)\hat{i} + \left(\hat{j} \cdot \vec{a}\right)\hat{j} + \left(\hat{k} \cdot \vec{a}\right)\hat{k}\right].$

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Now, $\hat{i}.\vec{a} = \hat{i} \cdot \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$

$= a_1 i^2 + a_2\left(\hat{i} \cdot \hat{j}\right) = a_3\hat{k}\left(\hat{i} \cdot \hat{k}\right)$

$= a_1(1) + a_2(0) + a_3(0) = a_1$

Similarly, $\hat{j} \cdot \hat{a} = a_2\ 7$ and $\hat{k} \cdot \vec{a} = a_3$.

$\therefore$ The given expression $= 3\vec{a} - \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$

$$= 3\vec{a} - \vec{a} = 2\vec{a}.$$

112.(A) $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right)$

$= \vec{r} \times \left(\vec{c} \times \vec{d}\right)$, where $\vec{r} = \vec{a} \times \vec{b} = \left(\vec{r} \cdot \vec{d}\right)\vec{c} - \left(\vec{r} \cdot \vec{c}\right)\vec{d}$

$= \left(\vec{a} \times \vec{b} \cdot \vec{d}\right)\vec{c} - \left(\vec{a} \times \vec{b} \cdot \vec{c}\right)\vec{d} \quad \left[\because \vec{r} = \vec{a} \times \vec{b}\right]$

$= \left[\vec{a}\ \vec{b}\ \vec{d}\right]\vec{c} - \left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{d}.$

113.(B) We have, $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a} \cdot \vec{b}\right)\vec{b}$

$= (4 - 2\beta - \sin\alpha)\, b + (\beta^2 - 1)c$ ... (1)

and $\left(\vec{c} \cdot \vec{c}\right)\vec{a} = \vec{c}$ ... (2)

where $\vec{b}$ and $\vec{c}$ are non-collinear vectors and $\vec{a}$, $\vec{b}$ are scalar.

From (2), $(\vec{c}\cdot\vec{c})\vec{a}\cdot\vec{c} = \vec{c}\cdot\vec{c}$

$\therefore\ \vec{a}\cdot\vec{c} = 1$ . . .

(3)

From (1), we get

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = (\vec{a}\cdot\vec{b})\vec{b}$

$= (4 - 2\beta - \sin\alpha)\vec{b} + (\beta^2 - 1)\vec{c}$

or $\quad \{1 + (\vec{a}\cdot\vec{b})\}\vec{b} - (\vec{a}\cdot\vec{b})c = (4 - 2\beta - \sin\alpha)\vec{b} + (\beta^2 - 1)\vec{c}$

$\Rightarrow \quad 1 + (\vec{a}\cdot\vec{b}) = 4 - 2\beta - \sin\alpha$ ... (4)

and $\quad \vec{a}\cdot\vec{b} = -(\beta^2 - 1)$ ... (5)

$\therefore \quad \sin\alpha = 1 + (1 - \beta)^2 \Rightarrow \beta = 1, \sin\alpha = 1$

i.e., $\alpha = \dfrac{\pi}{2} + 2n\pi,\ n \in I.$

114.(C) Let $\alpha = (x, 3, -7)$ and $\vec{b} = (x, -x, 4)$.

$\because \quad (\vec{a}, \vec{b})$ is acute

$\therefore \quad \vec{a}\cdot\vec{b} > 0 \Rightarrow x^2 - 3x - 28 > 0 \Rightarrow (x - 7)(x + 4) > 0$

$\Rightarrow$ x does not belong to (–4, 7)

$\Rightarrow \quad x \in R - (-4, 7)$.

Hence the interval in which x lies R – (–4, 7)

115.(A) We have, $\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|+2\vec{a}.\vec{b}$

$= (1)^2 + (1)^2 + 2\cdot1\cdot\cos60°$

$=1+1+1=2+1=3$

$\Rightarrow \quad \left|\vec{a}+\vec{b}\right|=\sqrt{3}>1.$

116.(A) Projections of $p\hat{i}+q\hat{j}+r\hat{k}=\vec{a}(\text{say})$ on x-axis = $\hat{a}\cdot\hat{i}$

$=\left(p\hat{i}+q\hat{j}+r\hat{k}\right)\cdot\hat{i}=p$

Similarly, projection of $p\hat{i}+q\hat{j}+r\hat{k}=\vec{a}$ on y-axis = $\vec{a}\cdot\hat{j}=q$

and projection of $p\hat{i}+q\hat{j}+r\hat{k}=\vec{a}$ on z-axis = $\vec{a}\cdot k=r$

$\therefore$ Sum of the projections of $\vec{a}$ on co-ordinates axes

= p + q + r = 2 + 3 +1 = 6

117.(D) $\because \vec{a}\perp\left(\vec{b}+\vec{c}\right)$

$\therefore \quad \vec{a}\cdot\left(\vec{b}+\vec{c}\right)=0\Rightarrow\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ ... (1)

Similarly $\vec{b}\perp(\vec{c}+\vec{a})=0\Rightarrow\vec{b}\cdot\vec{c}+\vec{b}\cdot\vec{a}=0$ ... (2)

and $\vec{c}\perp\left(\vec{a}+\vec{b}\right)=0\Rightarrow\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}=0$ ... (3)

Adding (1), (2), (3), we get

$2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$

Now, $\left(\vec{a}+\vec{b}+\vec{c}\right)^2=\vec{a}^2+\vec{b}^2+\vec{c}^2+2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 0$

$= (3)^2 + (4)^2 + (5)^2 = 50.$

Hence, $|a + b + c| = 5\sqrt{2}.$

118.(A) Here $\vec{a}$ and $\vec{c} = \vec{a} \times \vec{b}$ are non-collineary vectors.

$\therefore$ Let $\vec{b} = x\vec{a} + y(\vec{a} \times \vec{c})$ ... (1)

$\therefore$ $\beta = \vec{a} \cdot \vec{b} = \vec{a} \cdot [x\vec{a} + y(\vec{a} \times \vec{c})]$

$= x|\vec{a}|^2 + y\vec{a} \cdot (\vec{a} \times \vec{c}) = x\vec{a}^2$

$\Rightarrow$ x = β/a^2.

And $\vec{c} = \vec{a} \times b = \vec{a} \times [x\vec{a} + y(\vec{a} \times \vec{c})]$

$= x\vec{a} \times \vec{a} + y\vec{a} \times (\vec{a} \times \vec{c})$

$= 0 + y(\vec{a} \cdot \vec{c})\vec{a} - y(\vec{a} \cdot \vec{a})\vec{c}$

$= y\left\{\vec{a} \cdot (\vec{a} \times \vec{b})\right\}a - y\vec{a}^2\vec{c} = -y\vec{a}^2\vec{c}$

$\Rightarrow$ y = $-1/a^2$

$\therefore$ from (1), $\vec{b} = (\beta\vec{a} - \vec{a} \times \vec{c}) / \vec{a}^2.$

119.(C) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$\therefore$ $\vec{a} \times (\vec{b} \times \vec{c}) = x\vec{a} + y\vec{b} + z\vec{c}$

$\Rightarrow$ $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = x\vec{a} + y\vec{b} + z\vec{c}$

$\Rightarrow \quad x = 0, y = \vec{a}\cdot\vec{c}$ and $z = -(\vec{a}\cdot\vec{b})$

$\Rightarrow \quad x = 0$

$$y = (2\hat{i}+\hat{j}+\hat{k})\cdot(2\hat{i}+\hat{j}+3\hat{k}) = 4+1+3 = 8$$

$$z = -[2(1)+1(0)+1(3)] = -5$$

Hence (x, y, z) = (0, 8, –5).

120.(A) Let $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$

Then $\vec{a}\times\hat{i}+2\vec{a}-5\hat{j} = 0$

$\Rightarrow \quad (x\hat{i}+y\hat{j}+z\hat{k})\times\hat{i}+2(x\hat{i}+y\hat{j}+z\hat{k})-5\hat{j} = 0$

$\Rightarrow \quad -y\hat{k}+z\hat{j}+2(x\hat{i}+y\hat{j}+z\hat{k})-5\hat{j} = 0$

$\Rightarrow \quad (2x)\hat{i}+(z+2y-5)\hat{j}+(-y+2z)\hat{k} = 0$

$\Rightarrow \quad x = 0, z + 2y - 5 = 0, -y + 2z = 0$

$\Rightarrow \quad x = 0, y = 2, z = 1.$

$\therefore \quad \vec{a} = (0)\hat{i}+2\hat{j}+(1)\hat{k} = 2\hat{j}+\hat{k}.$